

# Making Cell Assemblies:

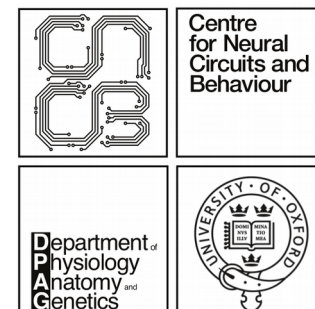
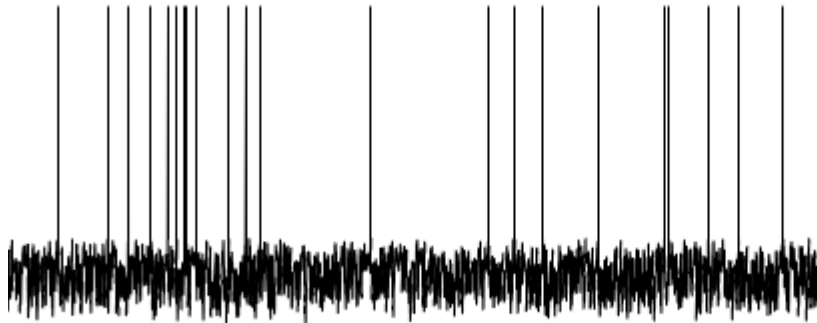
## What can we learn about plasticity from spiking neural network models?

Friedemann Zenke

EP Cognitive and Behavioural Neuroscience Seminar

Tuesday, 17 October 2017

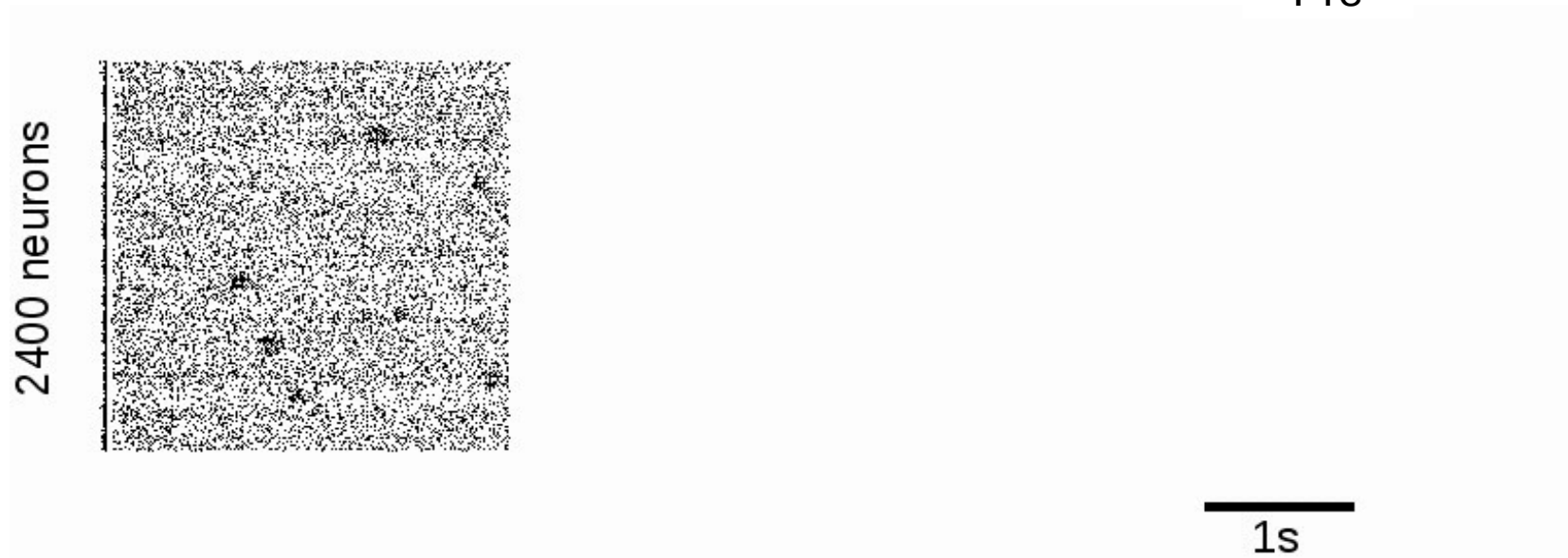
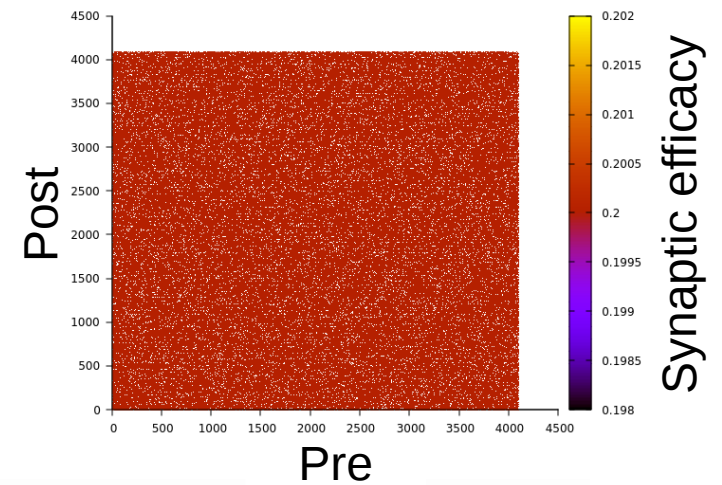
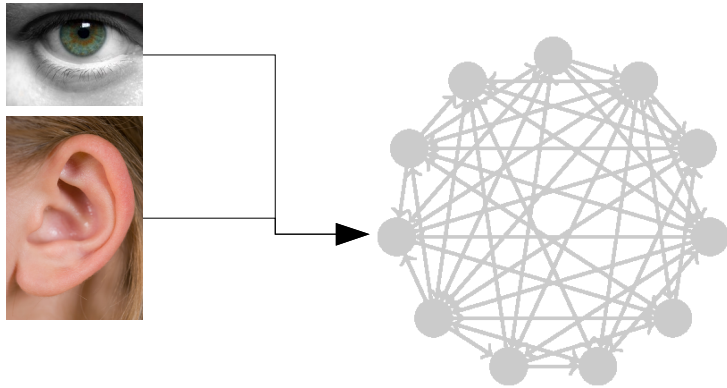
<https://fzenke.net>



# Outline

- Part 1: Cell assemblies and “The temporal paradox of Hebbian and homeostatic plasticity”
- Part 2: Supervised learning in spiking neural networks starting from a cost function

# Network activity in an unstructured balanced network

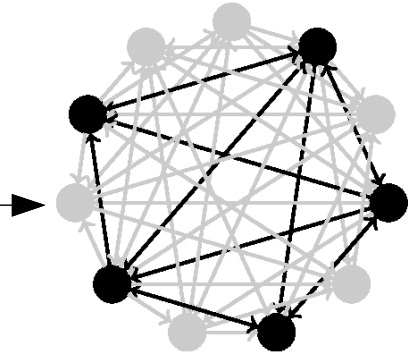
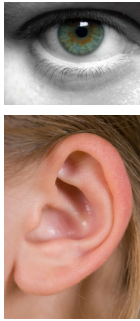


# Outline

- Part 1: Cell assemblies and “The temporal paradox of Hebbian and homeostatic plasticity”
- Part 2: Supervised learning in spiking neural networks starting from a cost function

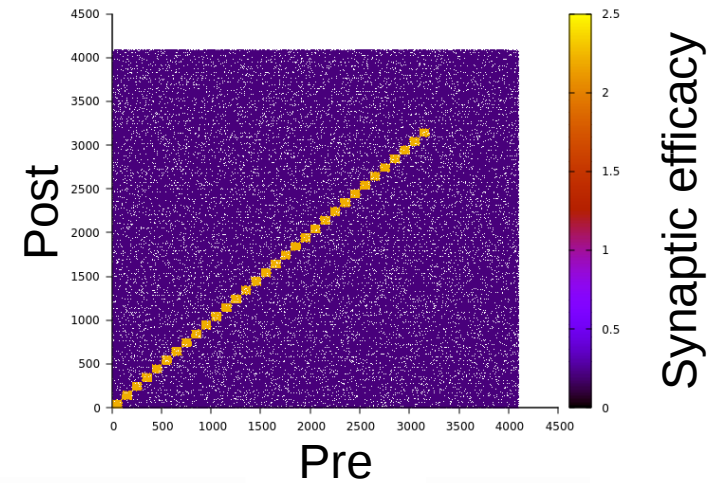


# Network activity in a balanced network with cell assemblies

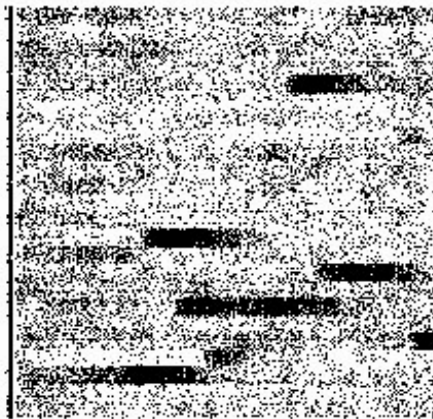


## Functions

- Associative
- Amplify
- Selective
- Working memory

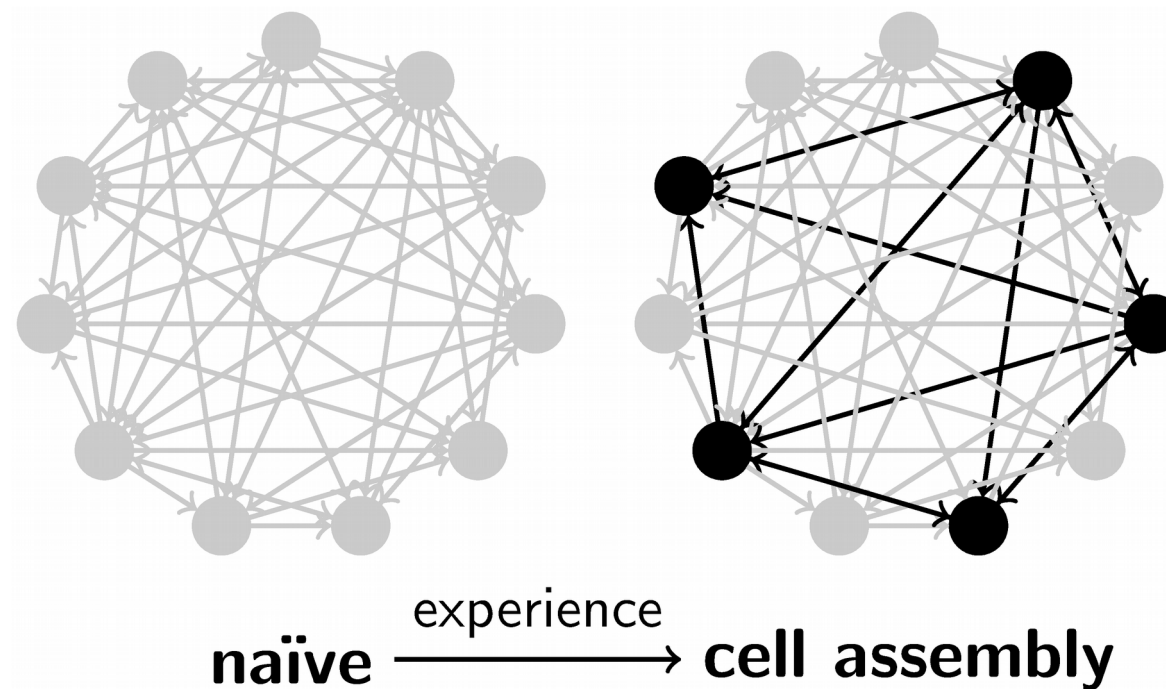


2400 neurons

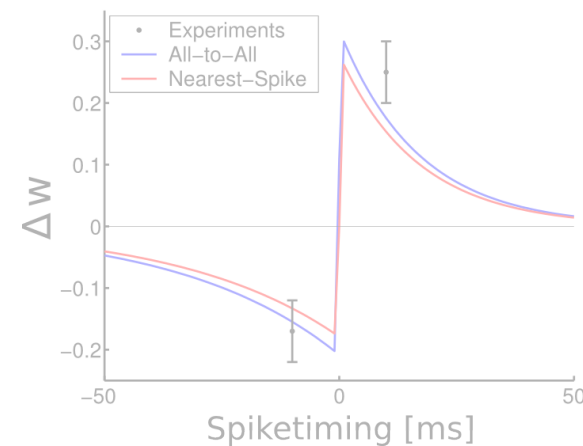
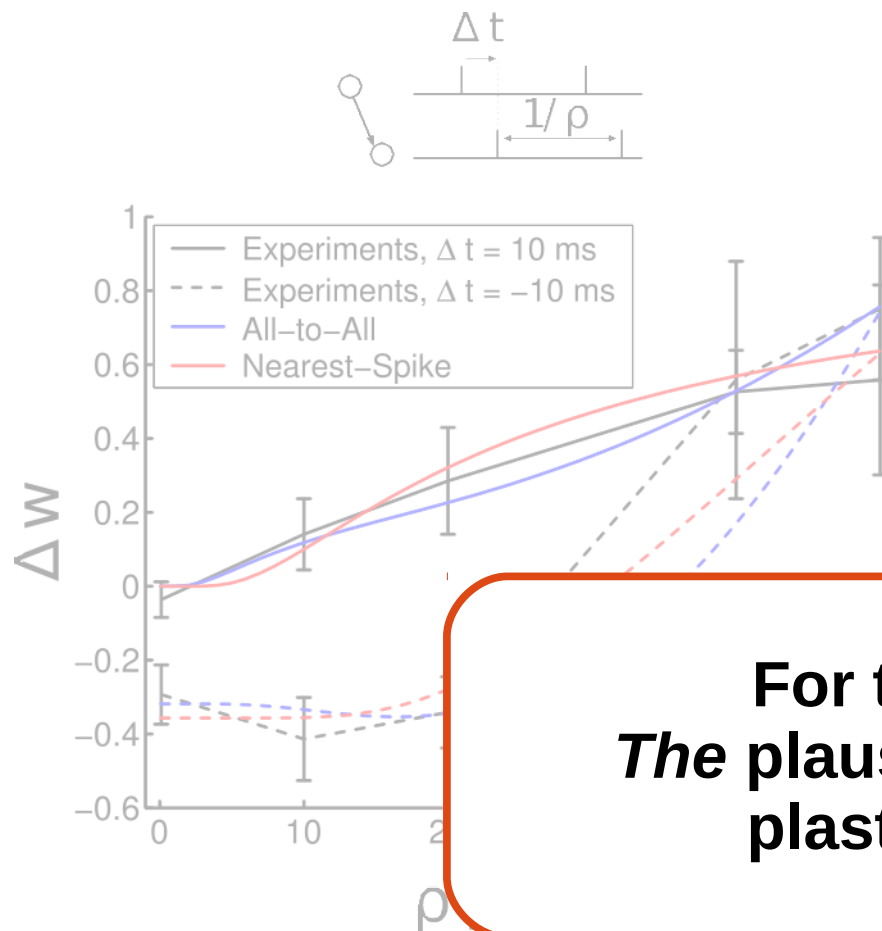


1s

# Cell assemblies are formed through Hebbian plasticity?



# Plasticity model: Triplet STDP



- Reproduces experiments

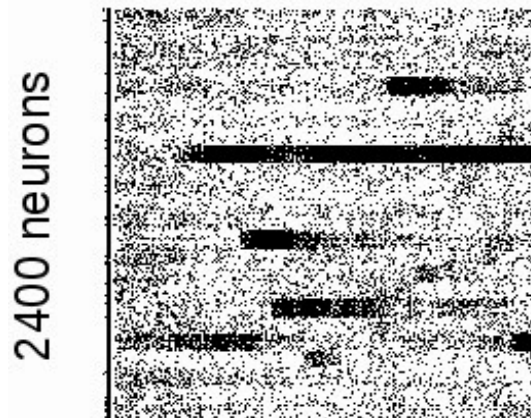
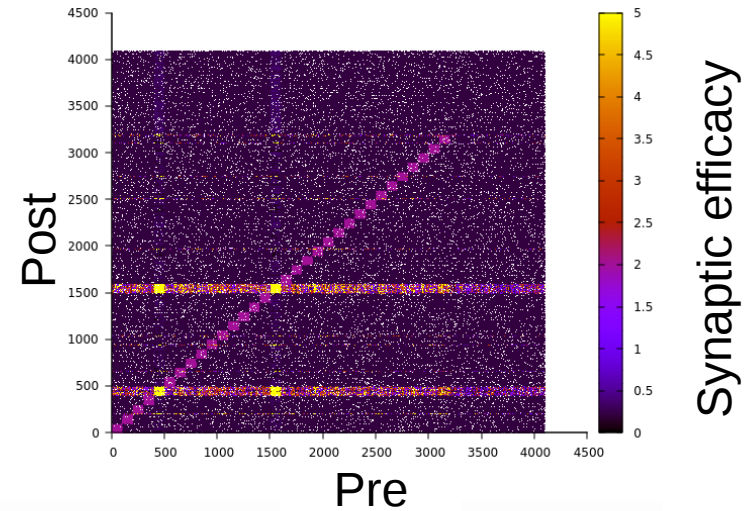
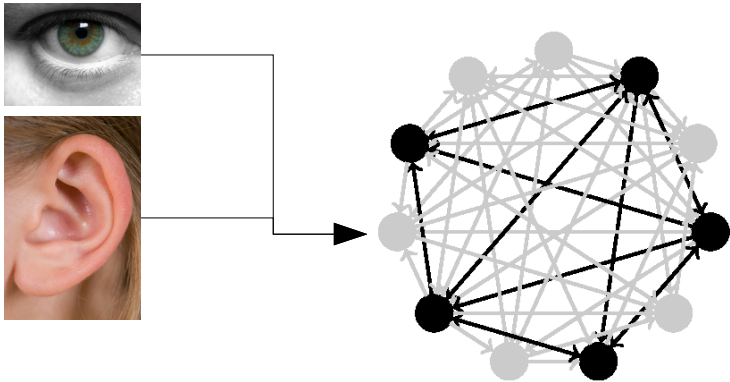
**For this talk:**  
***The plausible Hebbian***  
**plasticity rule**

Pfister & Gerstner (2006)

Experiments: Sjöström et al. (2001)

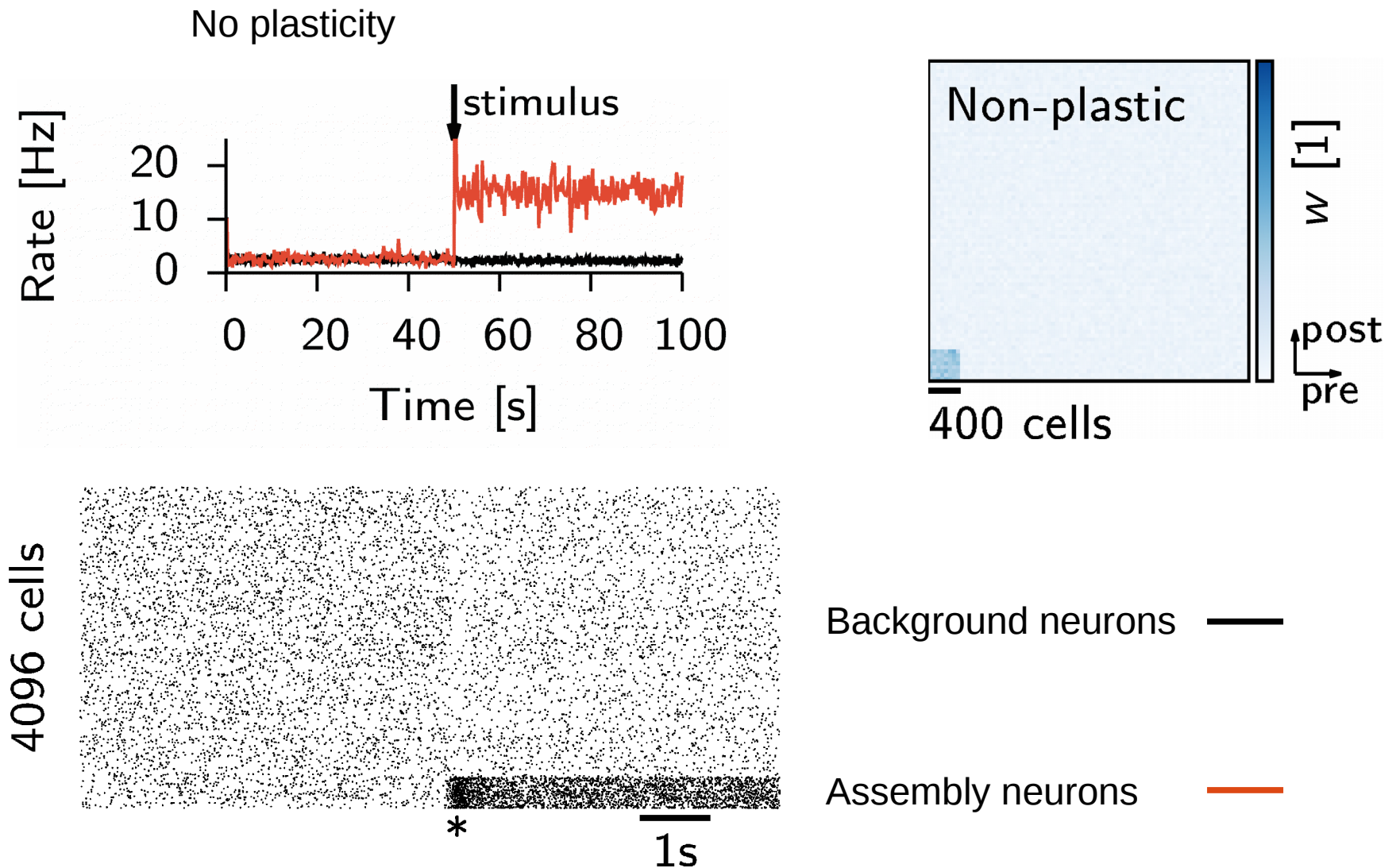
F. Zenke - fzenke.net

# Cell assemblies and Hebbian plasticity → Positive feedback loop

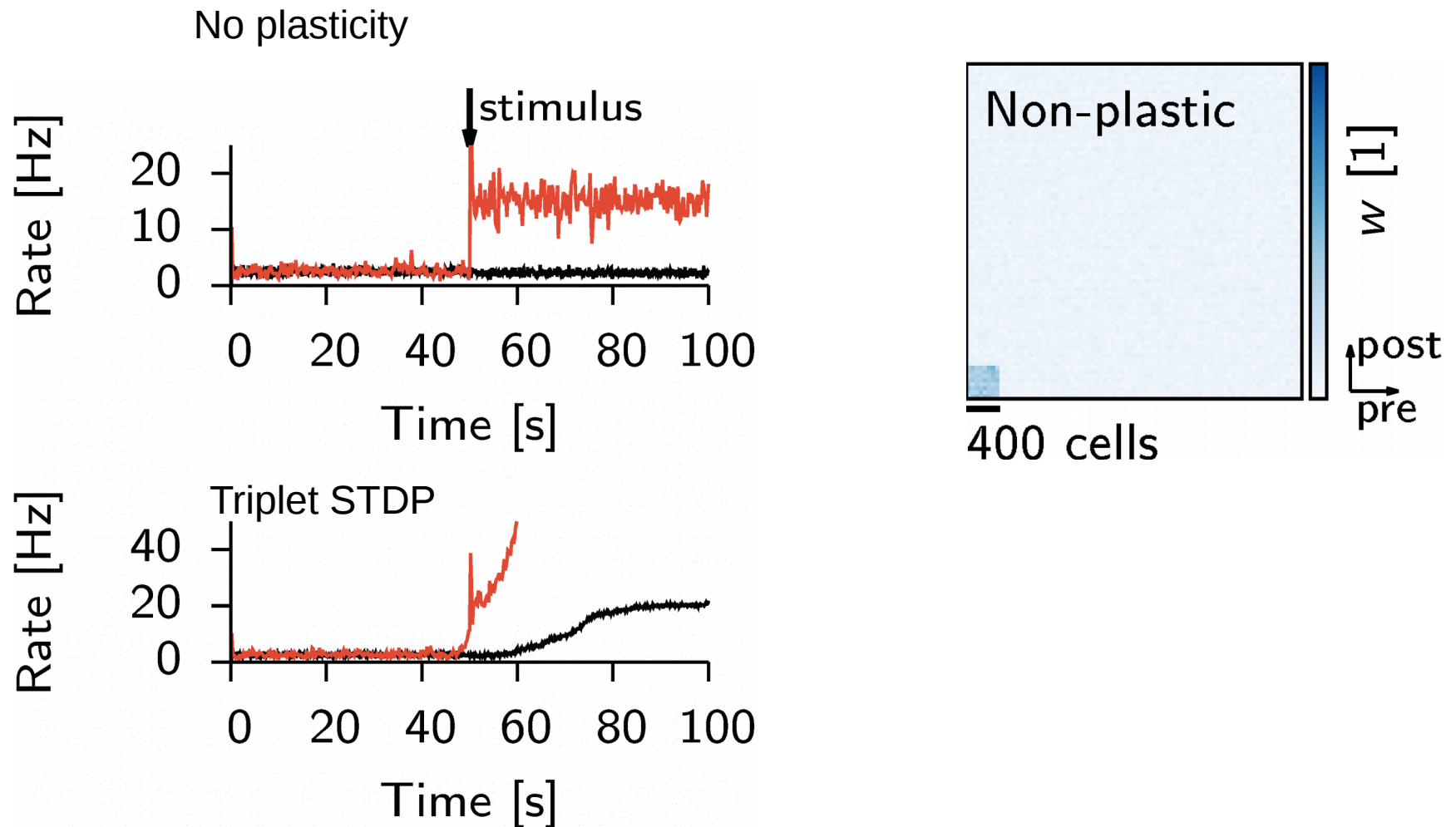


1s

# Bistable activity states are unstable in conjunction with triplet STDP

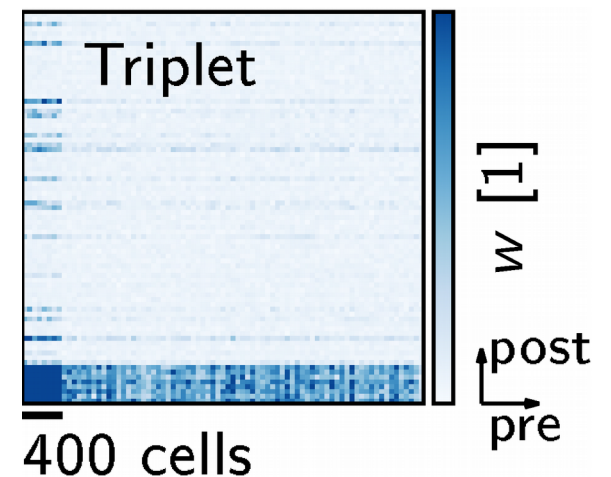
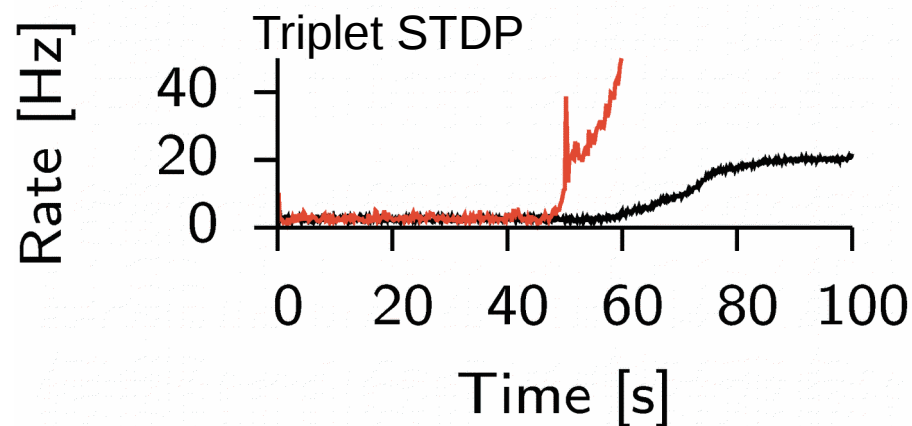
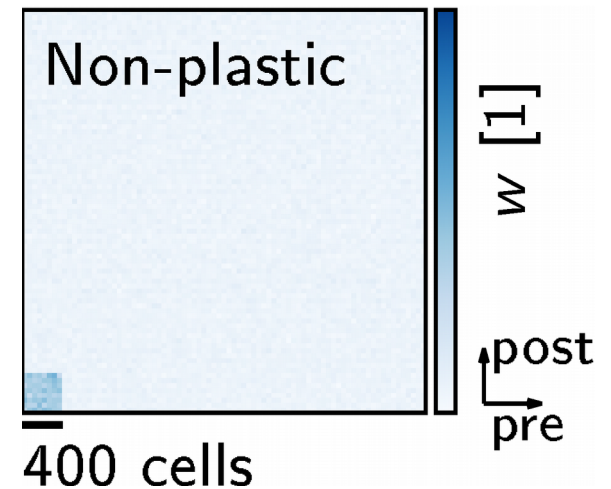
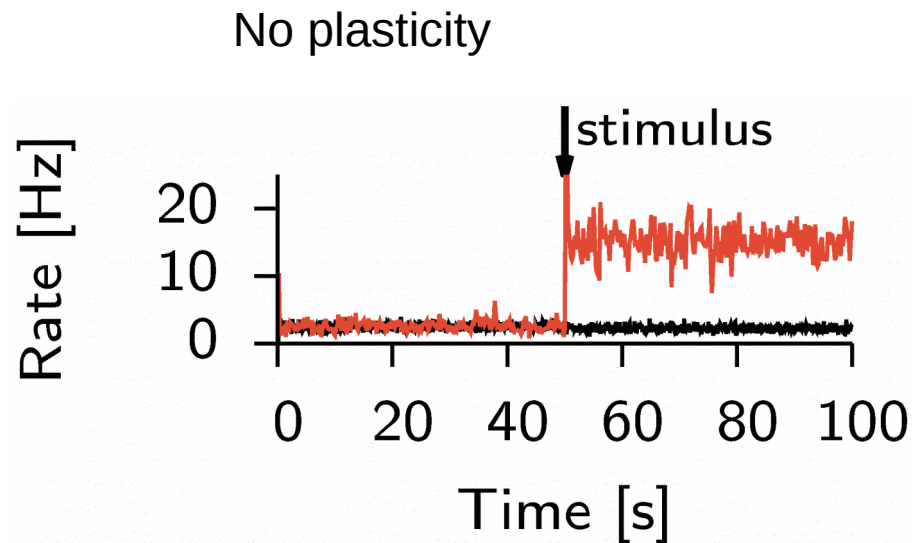


# Bistable activity states are unstable in conjunction with triplet STDP

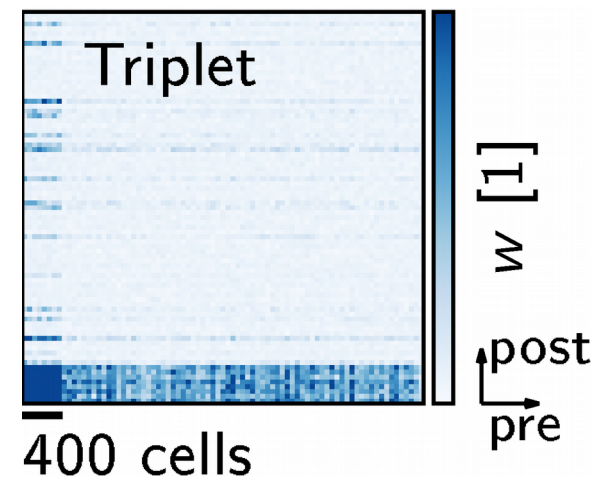
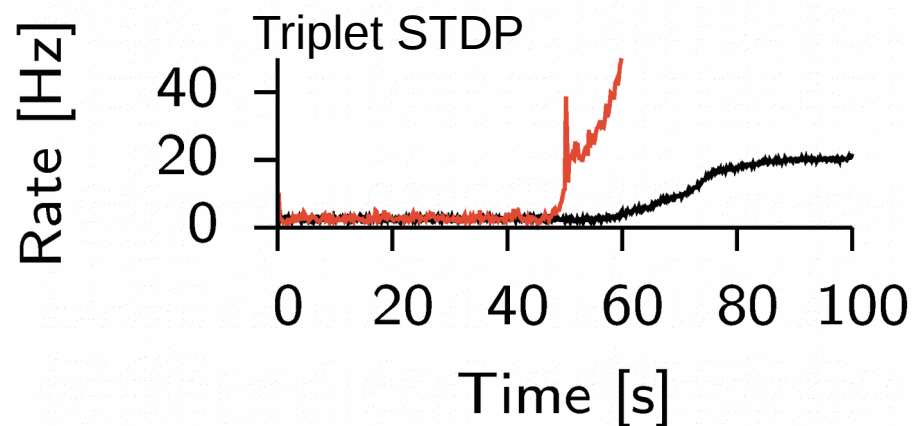
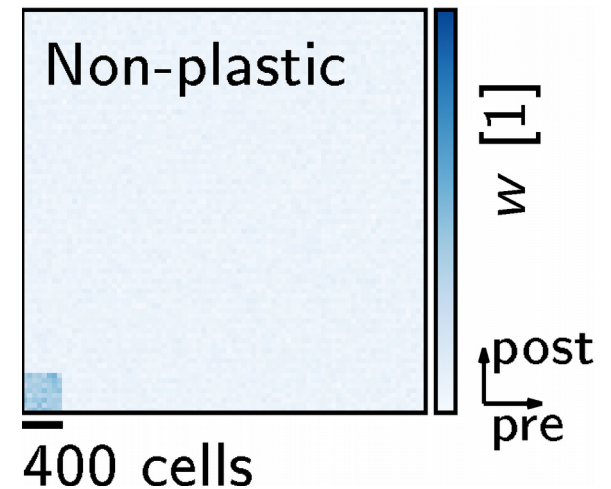
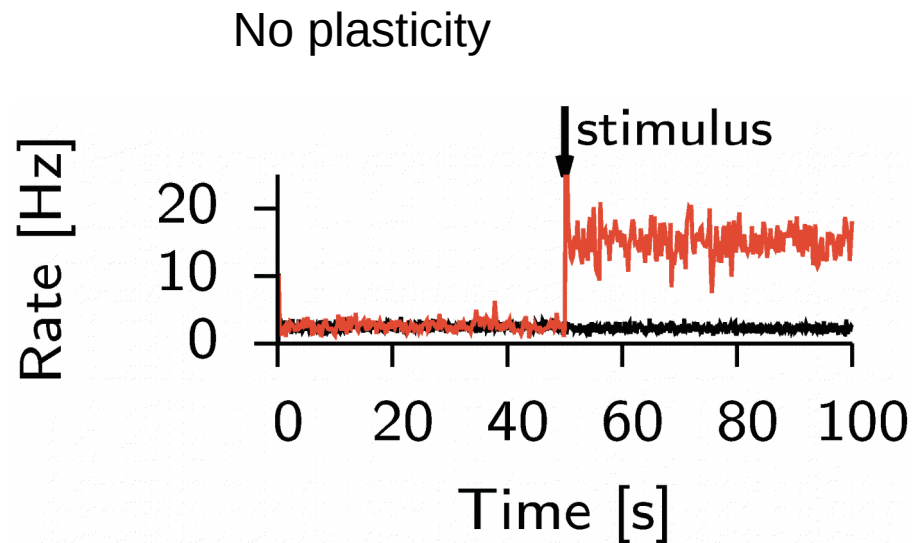




# Bistable activity states are unstable in conjunction with triplet STDP

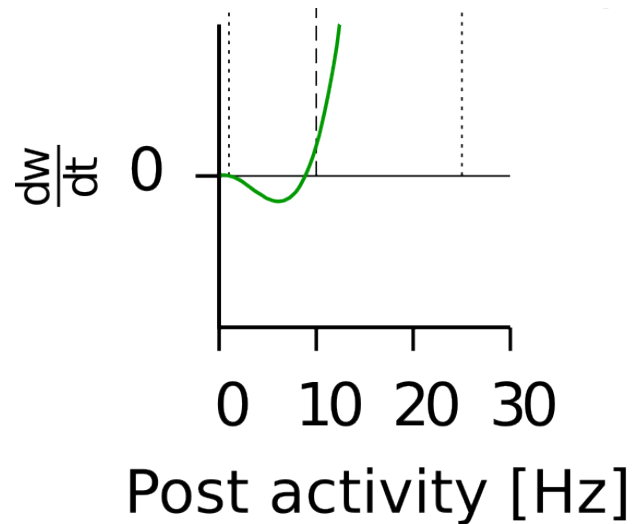


# Bistable activity states are unstable in conjunction with triplet STDP



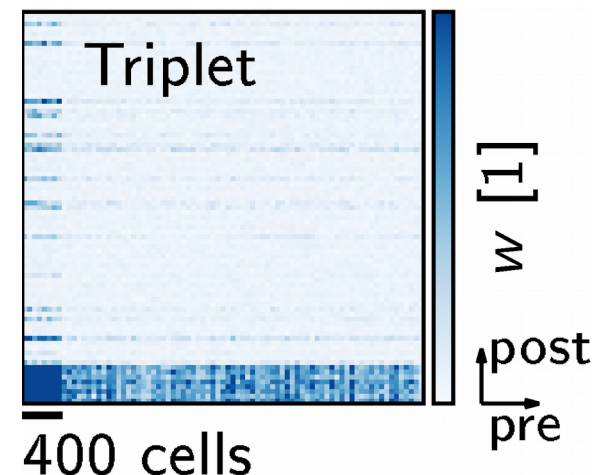
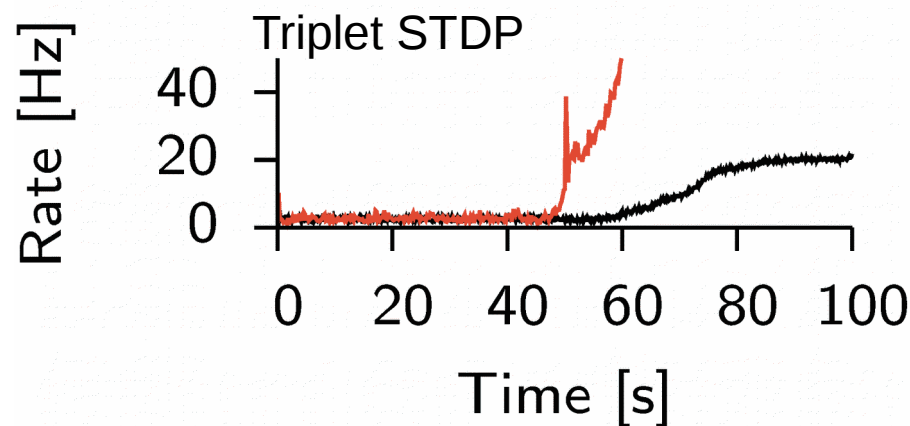


# Bistable activity states are unstable in conjunction with triplet STDP

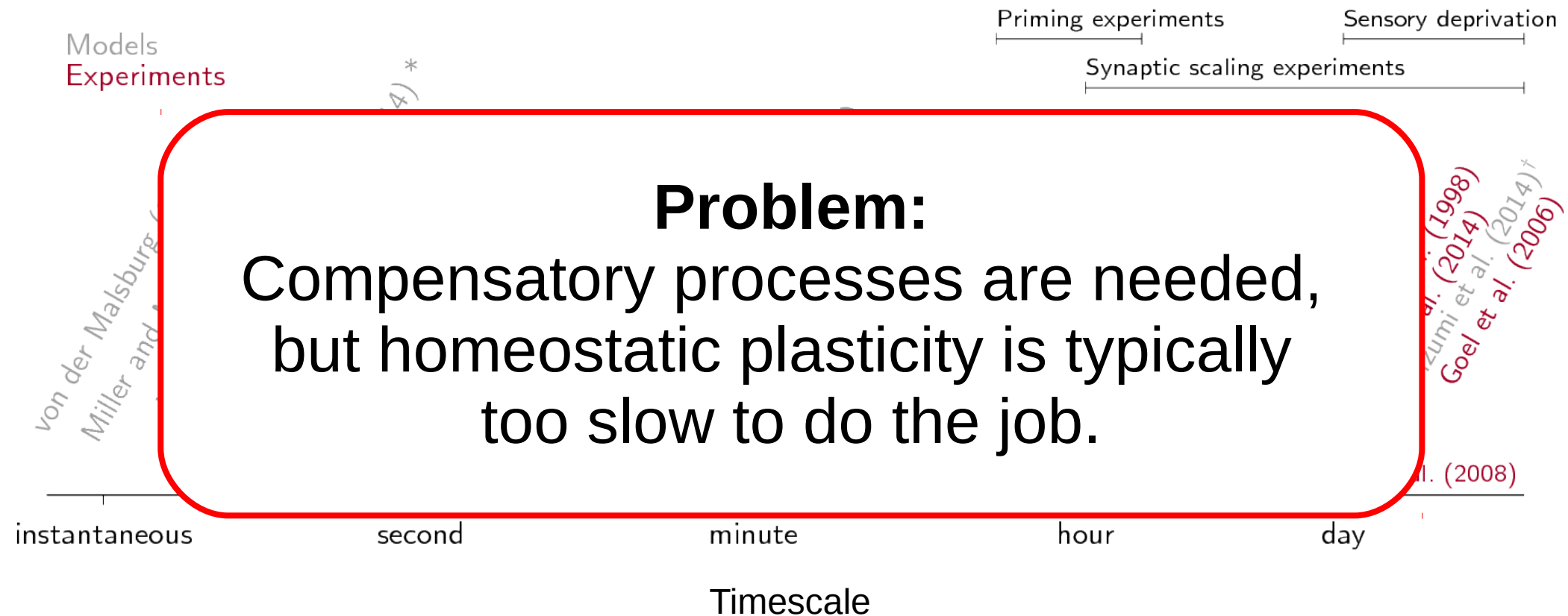


## Three important facts

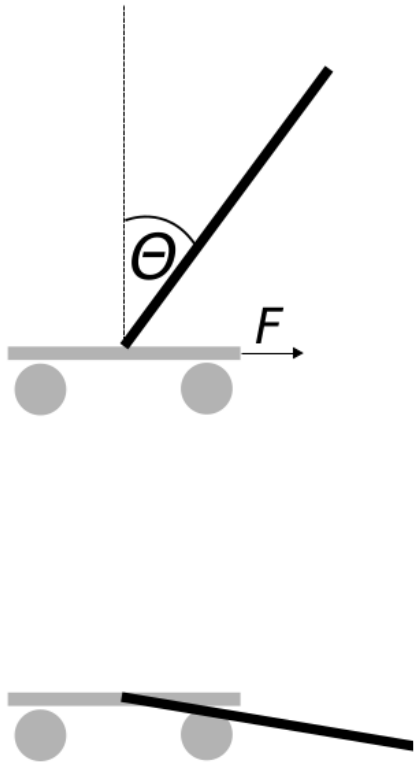
- Run-away potentiation introduces cross-talk and destroys memories
- Timescale of run-away
- Hebbian plasticity is unstable  
→ Need “homeostasis”



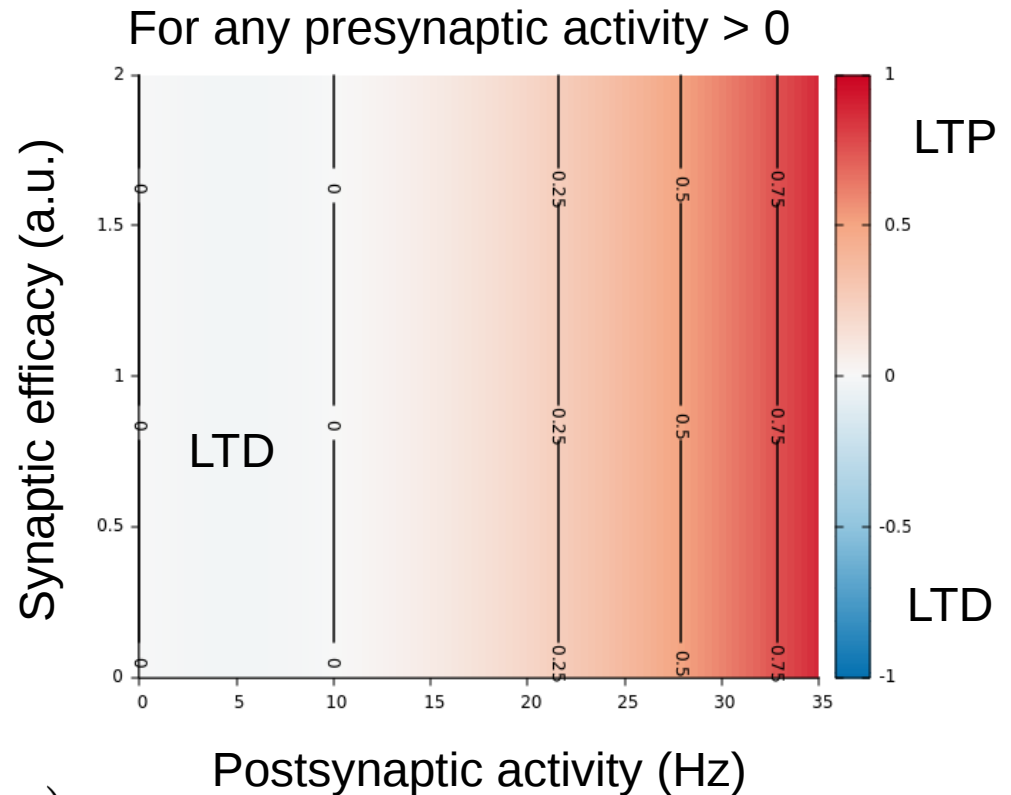
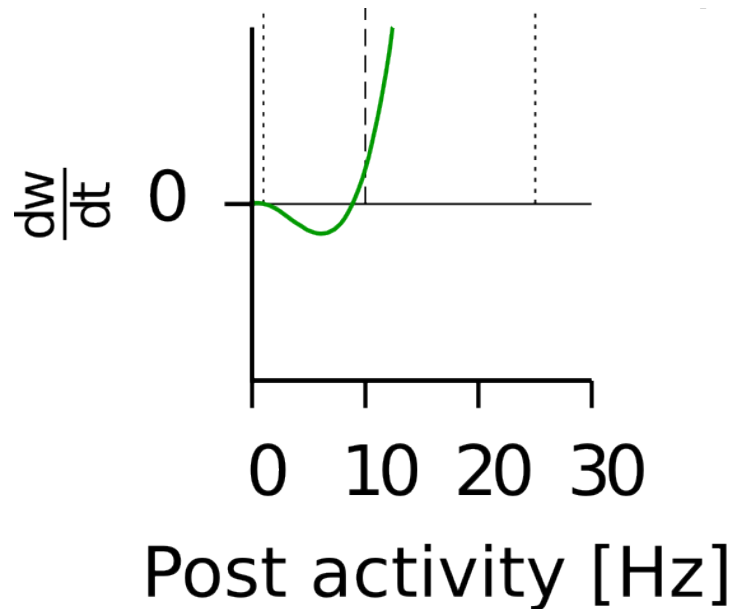
# The temporal paradox of Hebbian and homeostatic plasticity



# Control theory: Timescales of stabilizing processes needs to be matched to unstable system

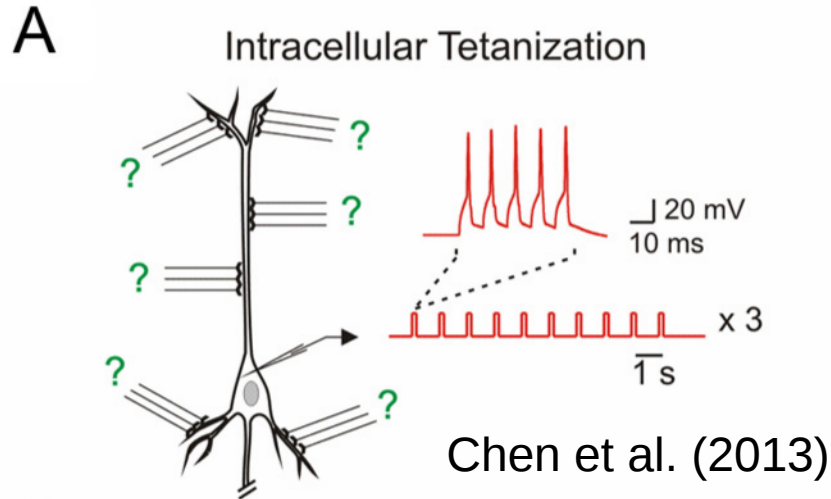


# Purely Hebbian learning with slow homeostasis is unstable



$$\frac{dw}{dt} \propto \underbrace{(\text{pre}) (\text{post})}_{\text{Purely Hebbian}} \underbrace{((\text{post}) - \kappa)}_{\text{Postsynaptic activation determines LTP/LTD}}$$

# Experimental evidence for heterosynaptic plasticity



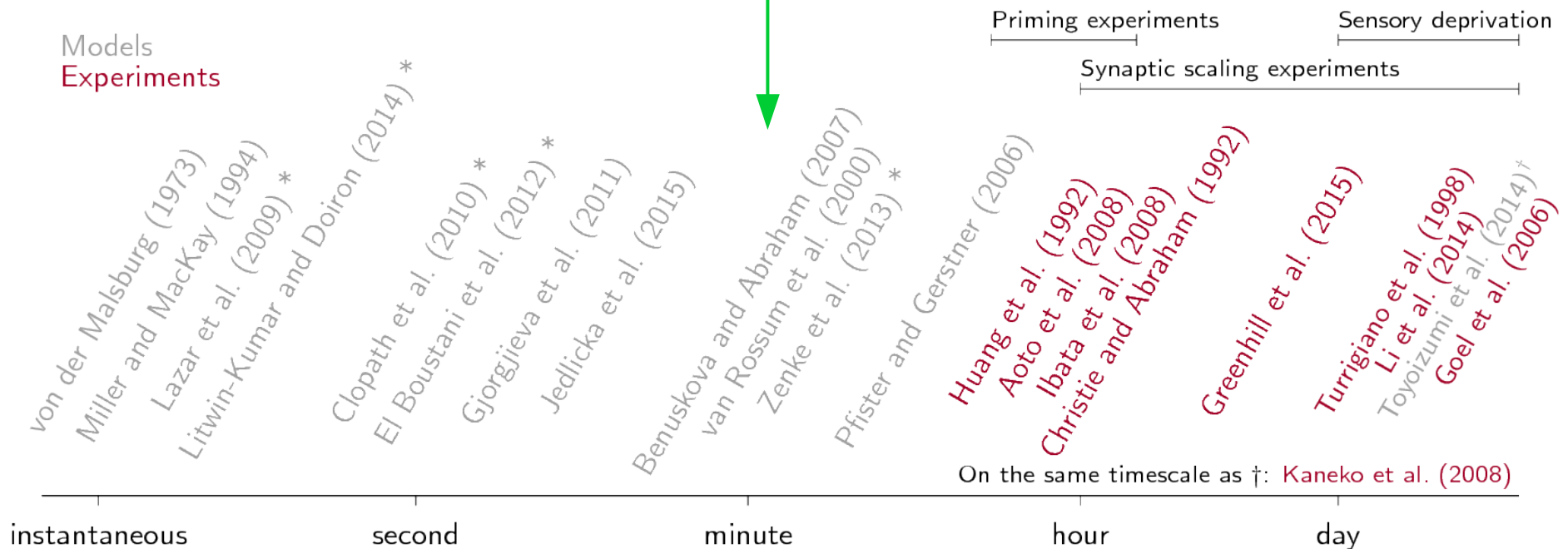
$$\frac{dw}{dt} \propto -\beta (\text{post})^4 (w - \tilde{w})$$

Zenke et al. (2015)

# Heterosynaptic plasticity acts on the right timescale

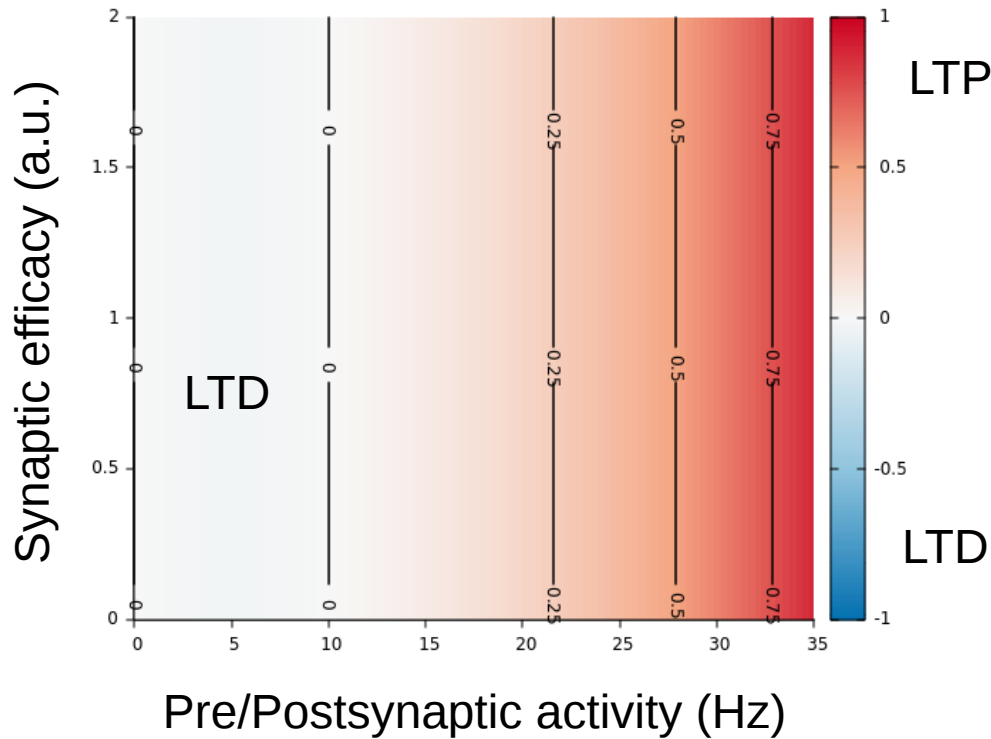
## Heterosynaptic plasticity

Royer and Paré, (2003), Chen et al., (2013), Oh et al., (2015)



# Non-Hebbian plasticity can act as rapid compensatory process

# Triplet STDP

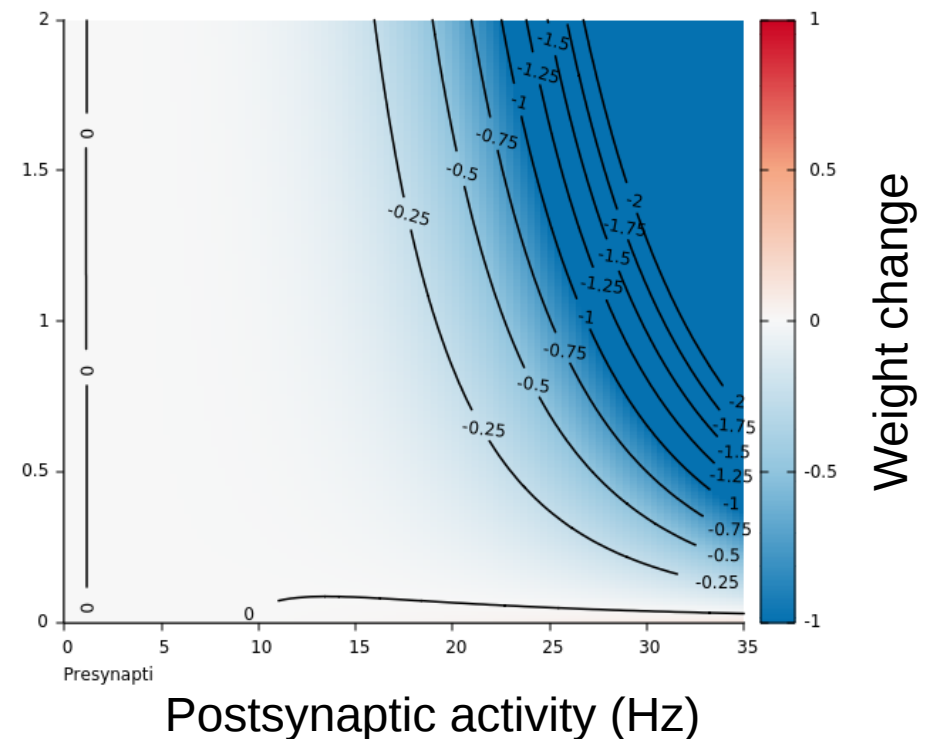
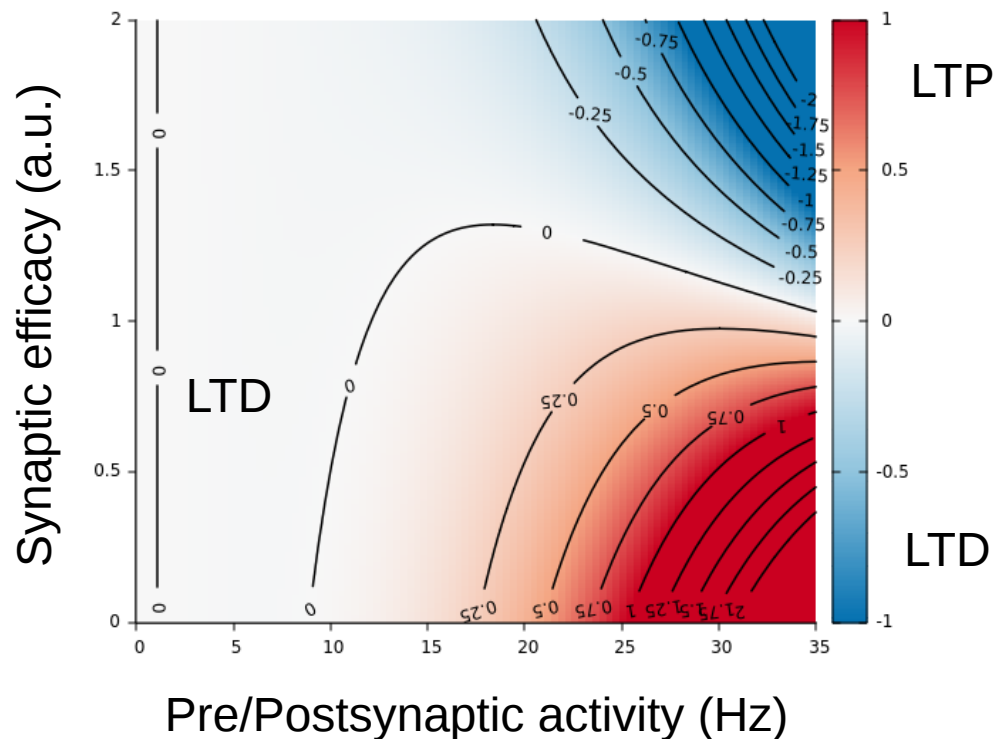


$$\frac{dw}{dt} \propto (\text{pre}) (\text{post}) ((\text{post}) - \kappa)$$

# Non-Hebbian plasticity can act as rapid compensatory process

Triplet STDP + heterosynaptic plasticity

Low presynaptic activity (1Hz)



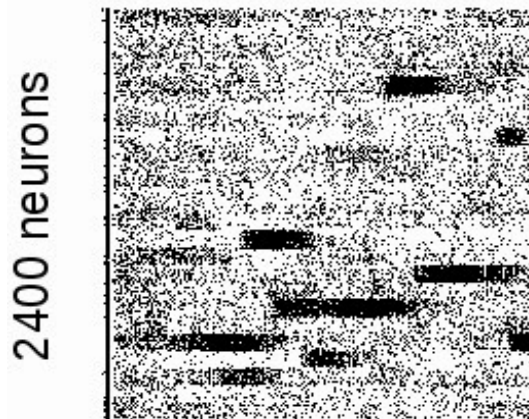
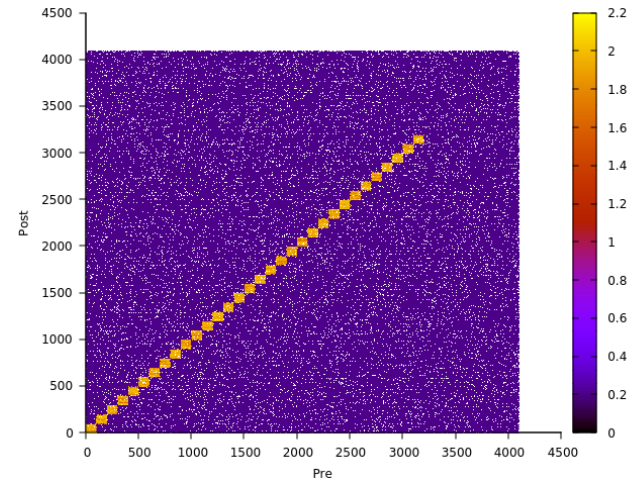
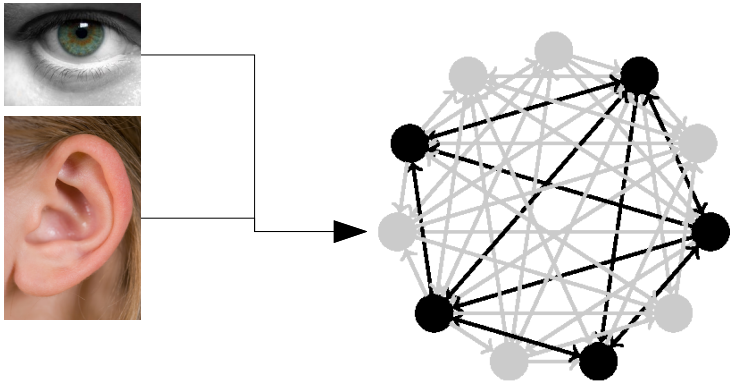
$$\frac{dw}{dt} \propto (\text{pre}) (\text{post}) ((\text{post}) - \kappa) - \beta (\text{post})^k (w - \tilde{w})$$

Heterosynaptic plasticity`

- Local
- Non-Hebbian
- Stabilizing



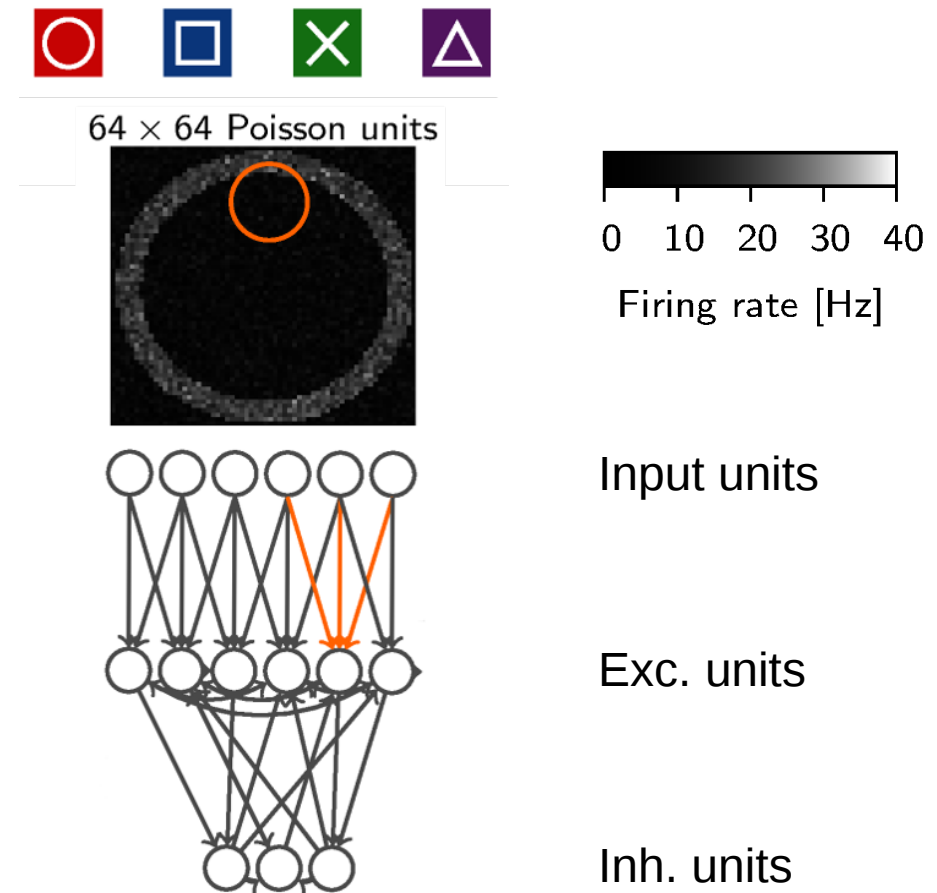
# Rapid compensatory processes can stabilize plastic network simulations

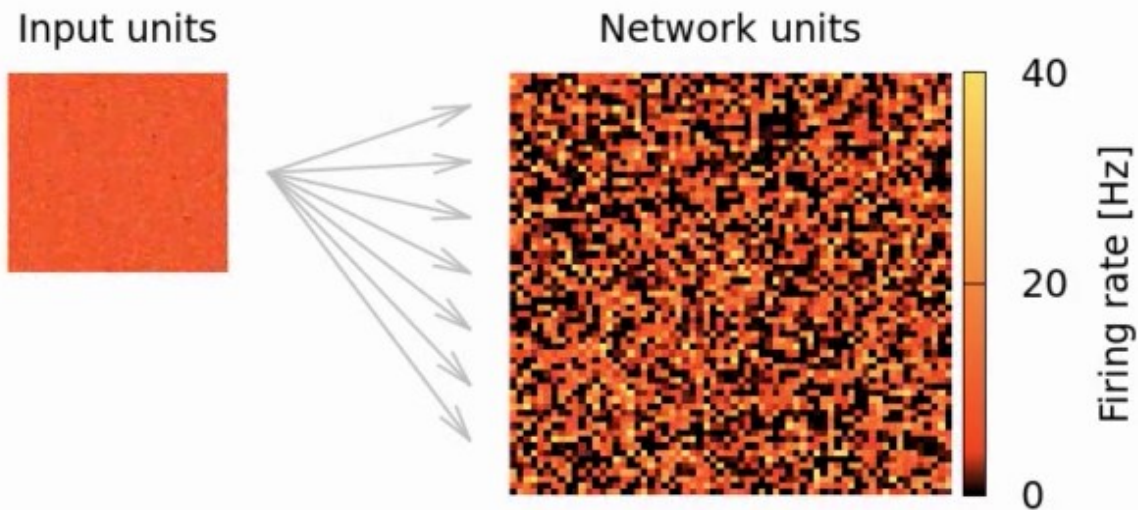


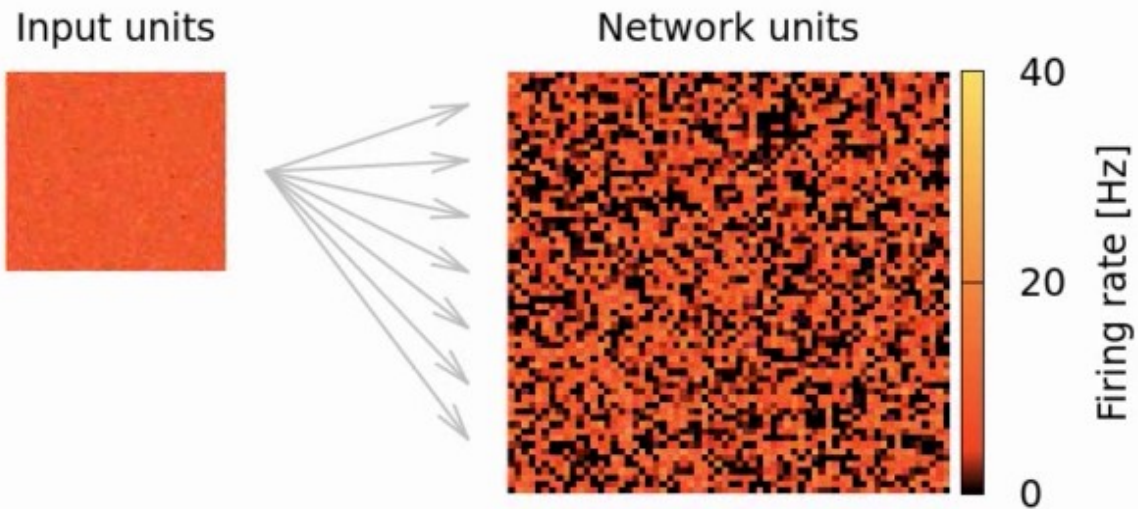
1s

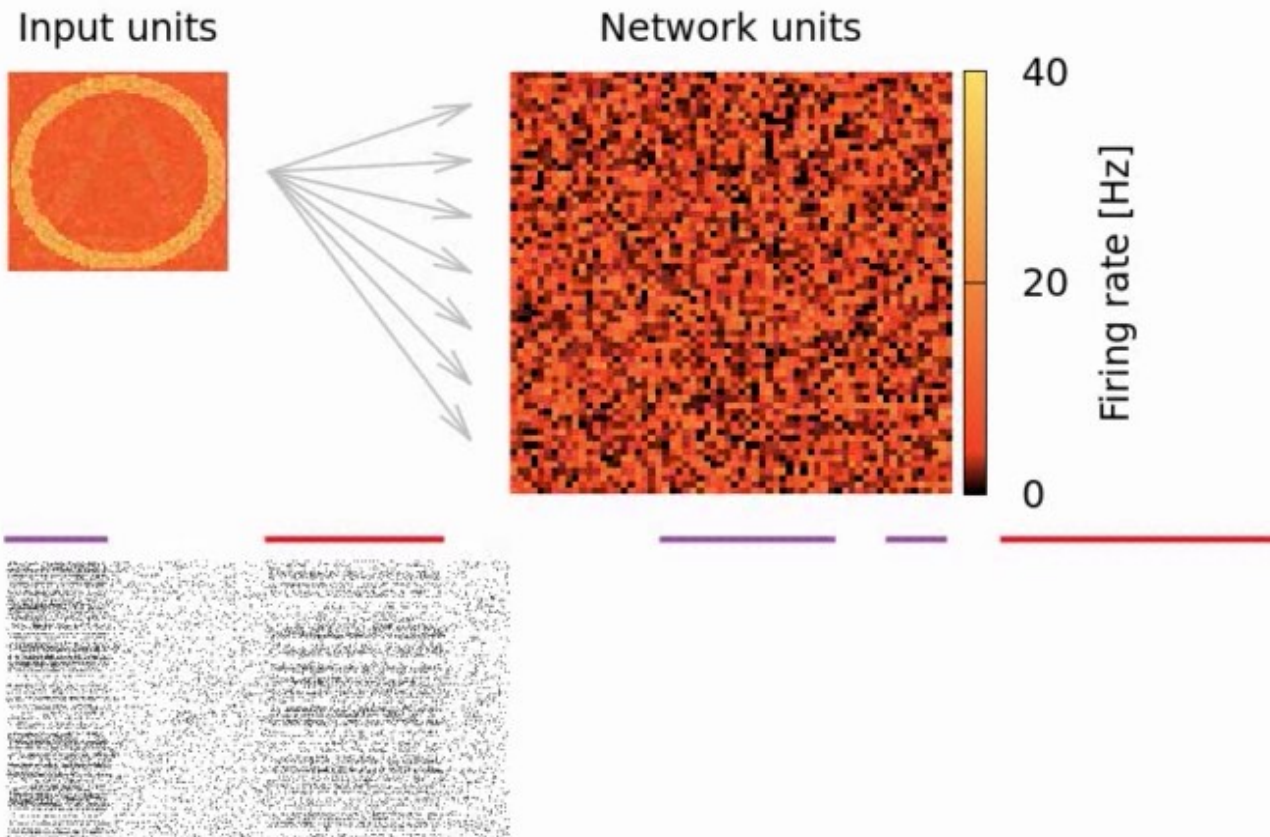
# Orchestrated forms of plasticity in a recurrent network model

- Plasticity
  - Triplet STDP
  - Heterosynaptic plasticity
  - Transmitter triggered plasticity
  - Inhibitory plasticity
  - Consolidation:  $\tilde{w}(t)$
- Spiking network
  - 4096 excitatory IF
  - 1024 inhibitory IF
  - Conductance based synapses
  - Short term plasticity
  - Spike triggered adaptation
  - Random sparse connectivity

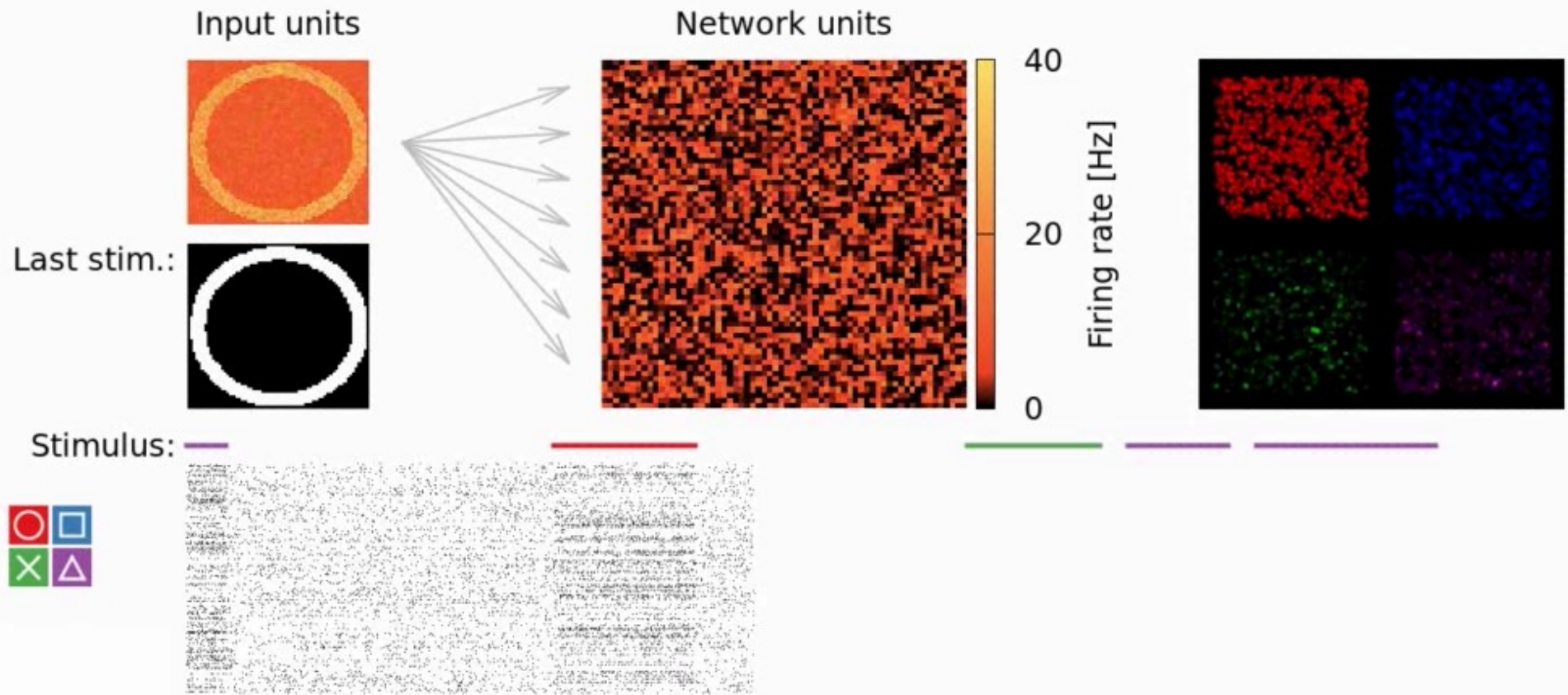


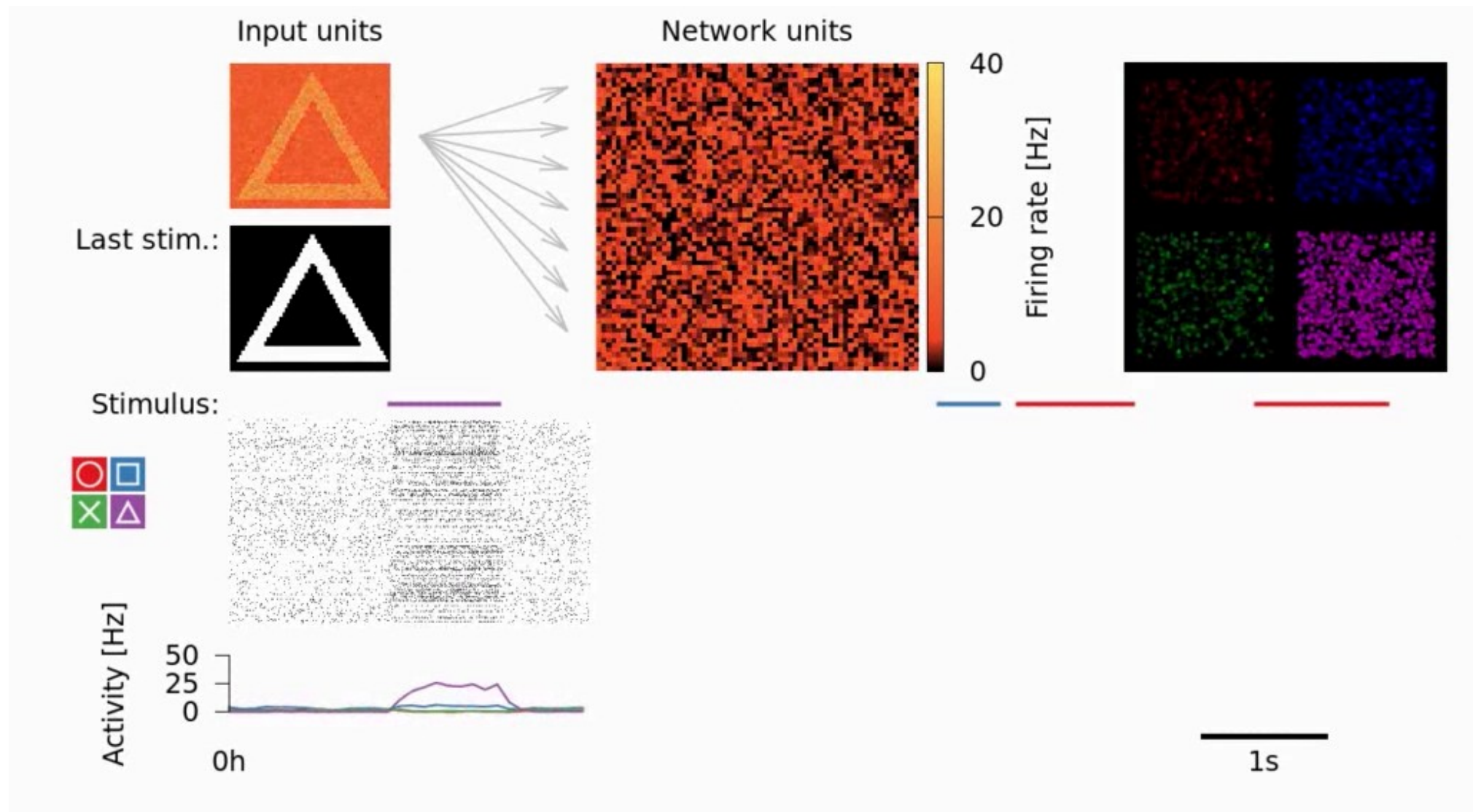




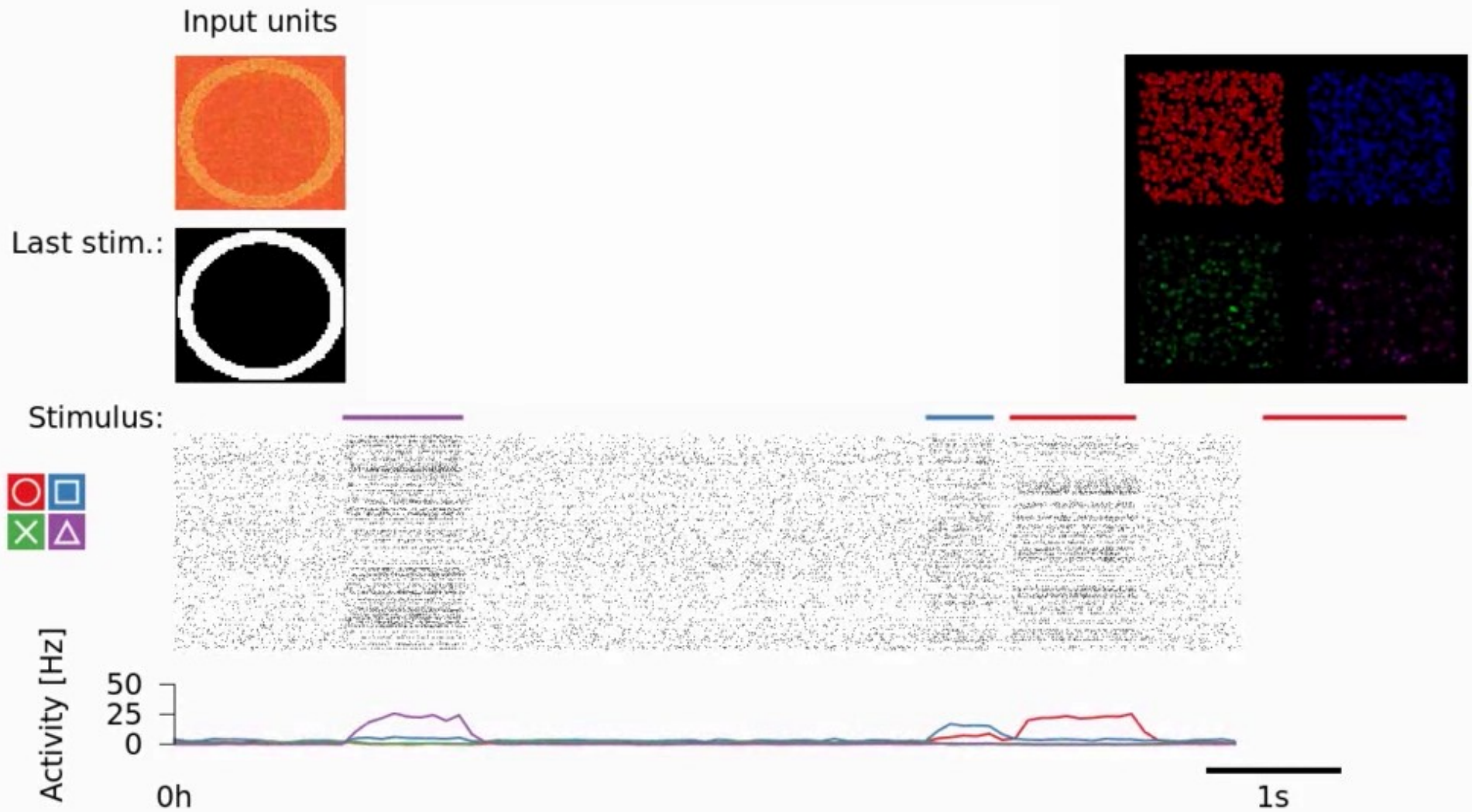








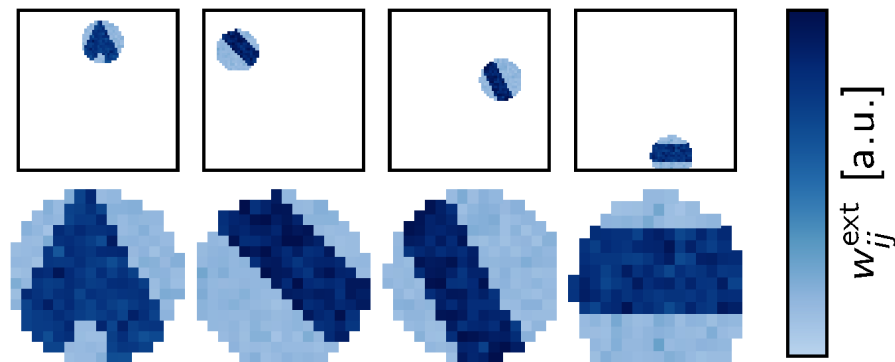
> External stimuli



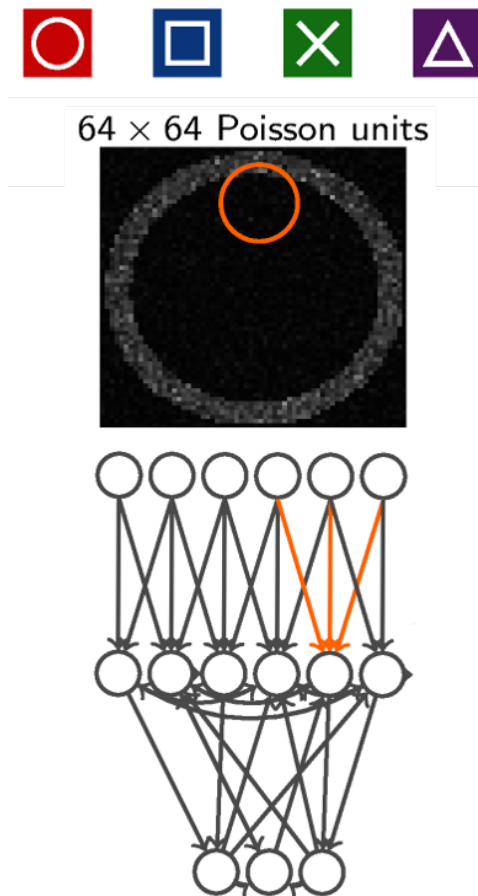
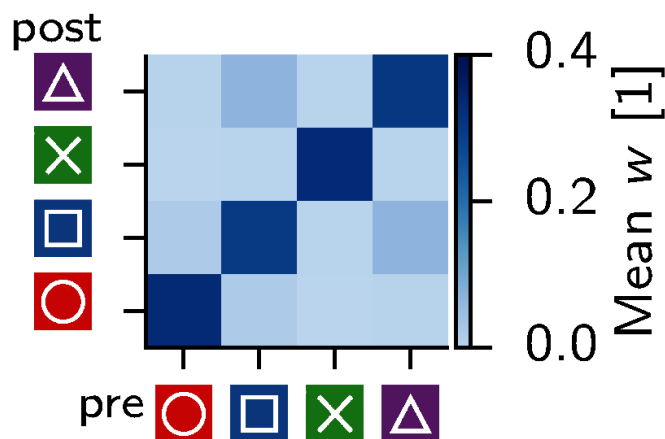


# Receptive fields are refined and cell assemblies are formed

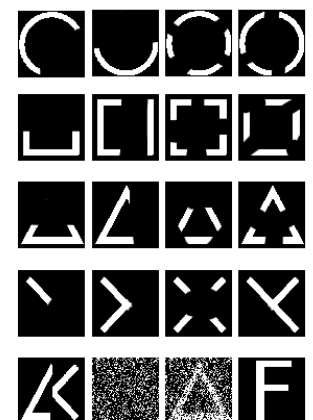
Input connections



Recurrent connections



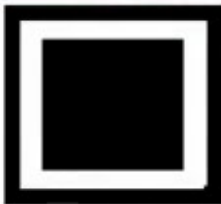
Distorted cues



Input units



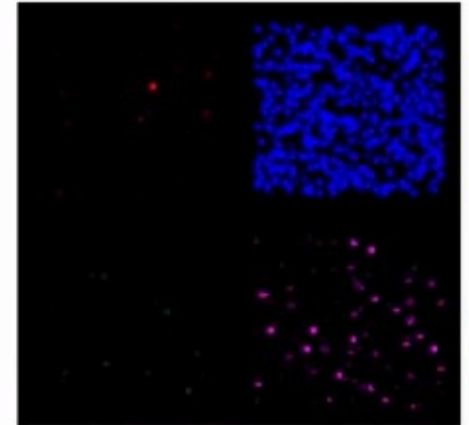
Last stim.:



Stimulus:



Activity [Hz]



1s

Distorted stimuli & recall (plasticity still active)

Zenke et al. (2015)

Input units



Last stim.:

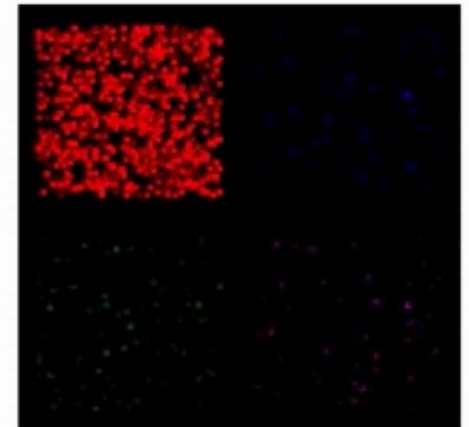


Stimulus:



Stimulus: A large block of text, likely a stimulus input, displayed in a small font.

Activity [Hz]



1s

Input units



Last stim.:

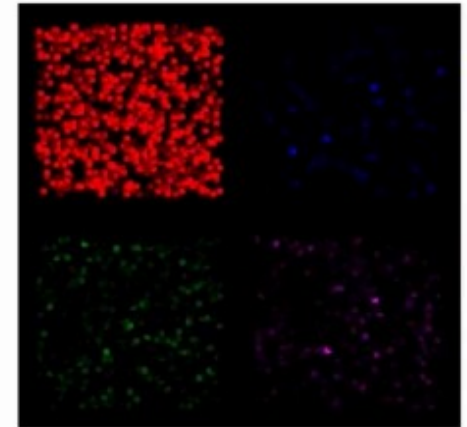
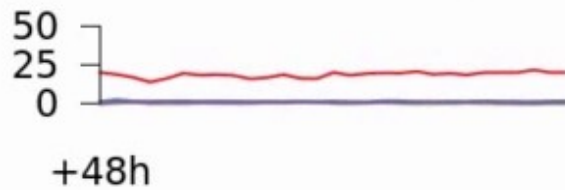


Stimulus:



Stimulus: A 2x2 grid of colored shapes: top-left is a red circle, top-right is a blue square, bottom-left is a green 'X', and bottom-right is a purple triangle.

Activity [Hz]



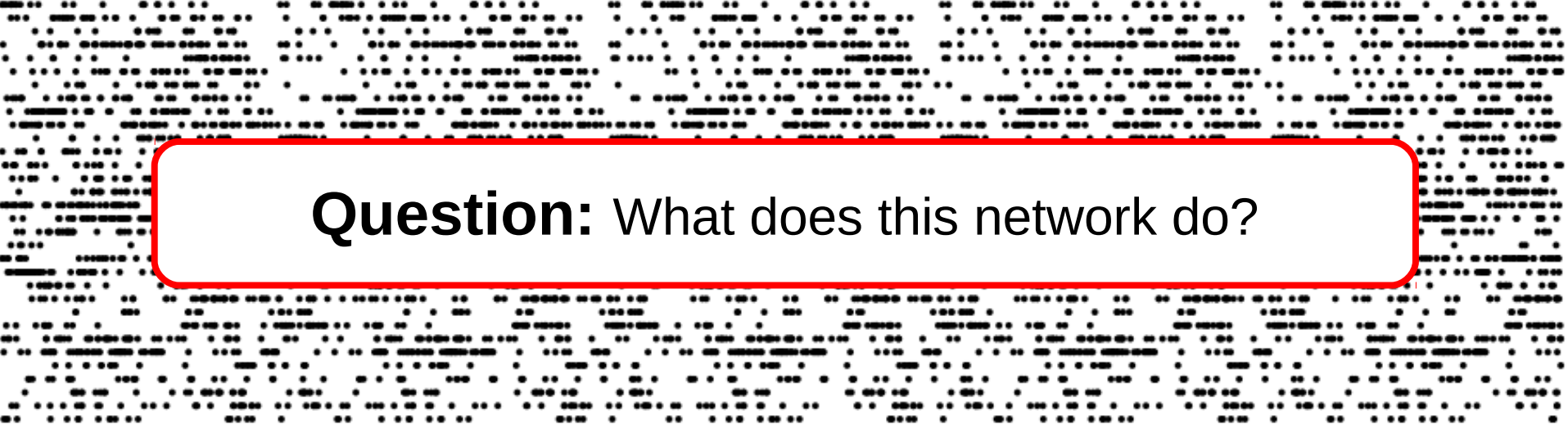
# Part 1: Summary & Conclusion

- Intrinsically stable learning rules necessary to create stable memories
- “Rapid compensatory processes”
  - Have to be on comparable timescale
  - Manifest as non-Hebbian contributions to plasticity

# Outline

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- Part 2: Supervised learning in spiking neural networks starting from a cost function

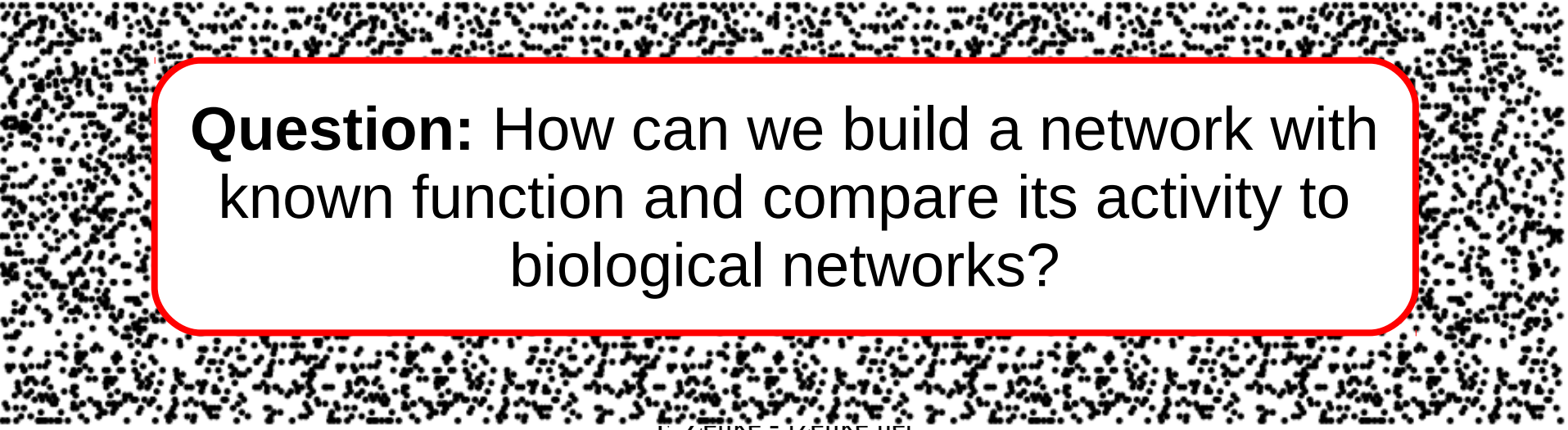
# Problem: Understanding neural activity



A raster plot showing neural activity over time. The plot consists of a grid of dots where each dot represents a spike from a specific neuron at a specific time. The dots are arranged in horizontal rows, with each row representing a different neuron. The overall pattern shows a mix of spontaneous activity and some more structured firing patterns.

**Question:** What does this network do?

1.00s



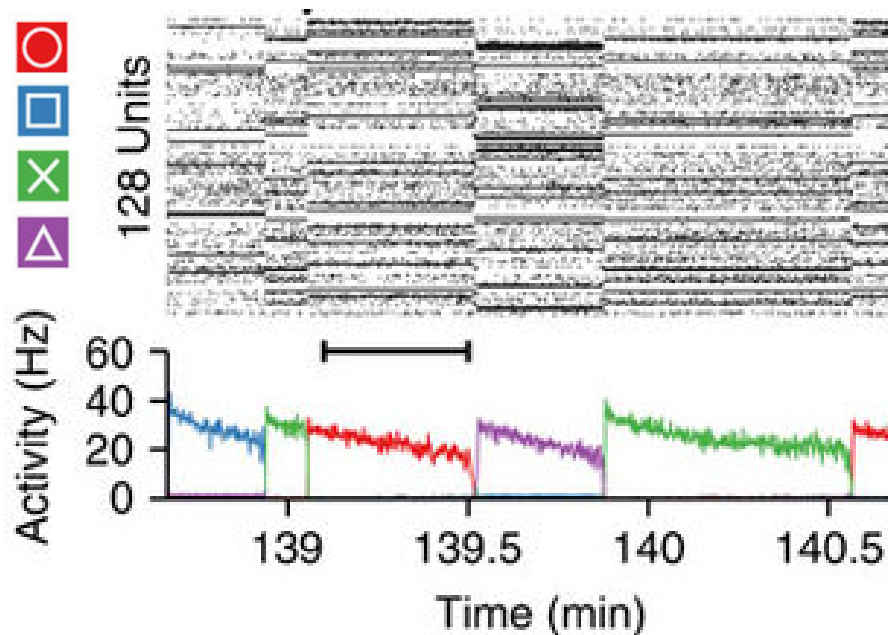
A raster plot showing neural activity over time, similar to the one above but with a much higher density of dots, indicating more frequent or synchronized firing across the network.

**Question:** How can we build a network with known function and compare its activity to biological networks?



# Our ability to engineer spiking neural networks which perform complex tasks is limited in comparison to rate-based models

## Computational neuroscience



Zenke et al (2015)

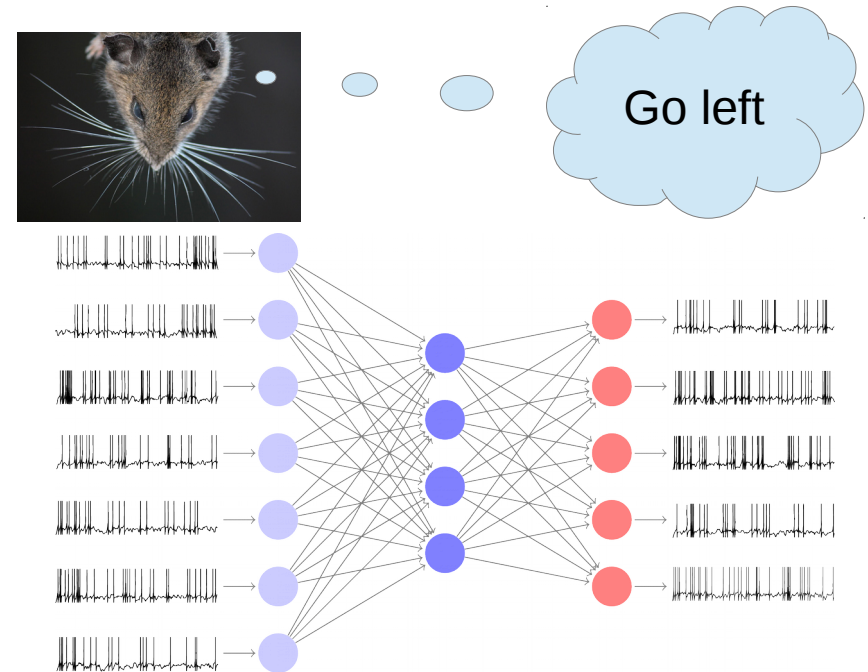
## Deep learning



Krizhevsky et al. (2012)

# Desiderata

- Spiking network which solves complex task
- Use spike timing
- Algorithm which could be conceivably

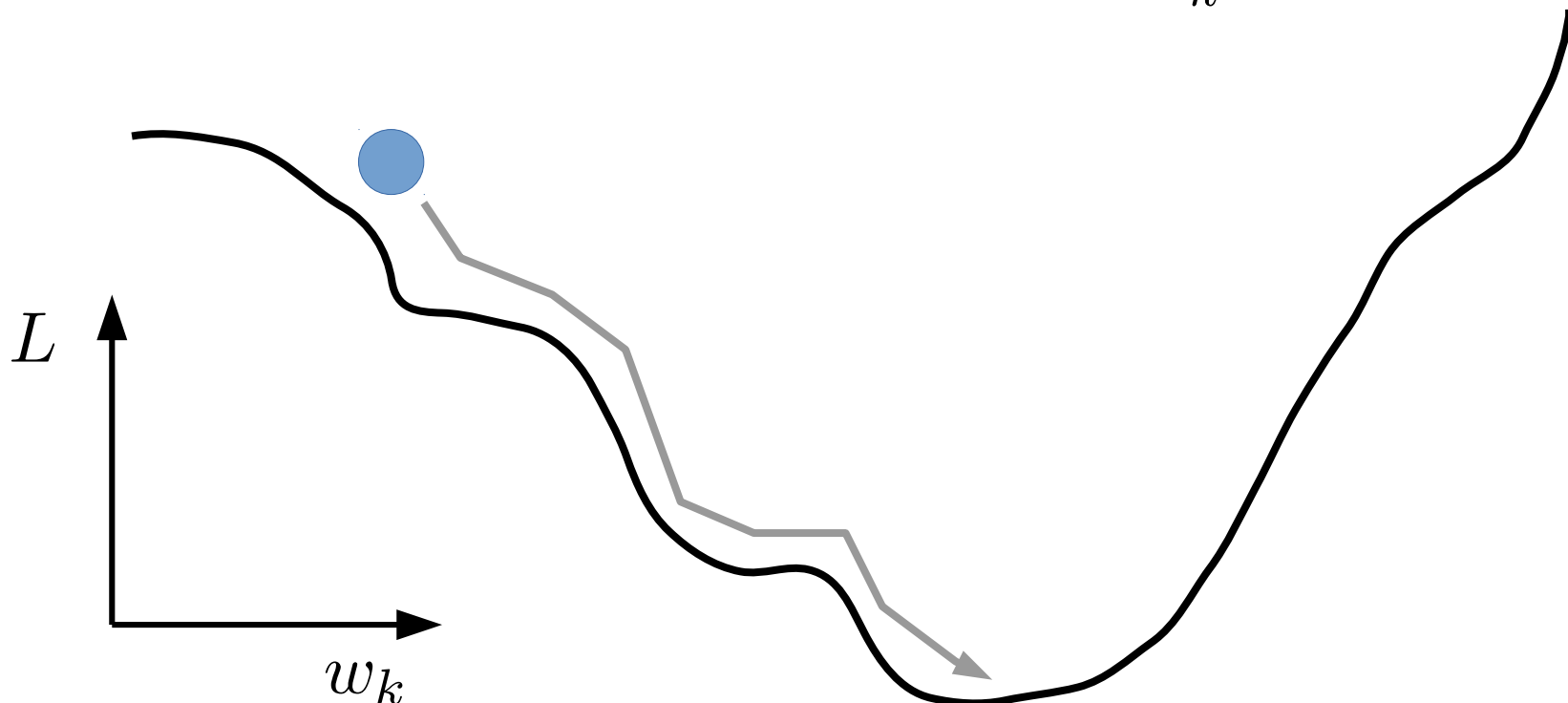


## Aim

Get spiking networks to do something interesting,  
by starting from an objective function approach.

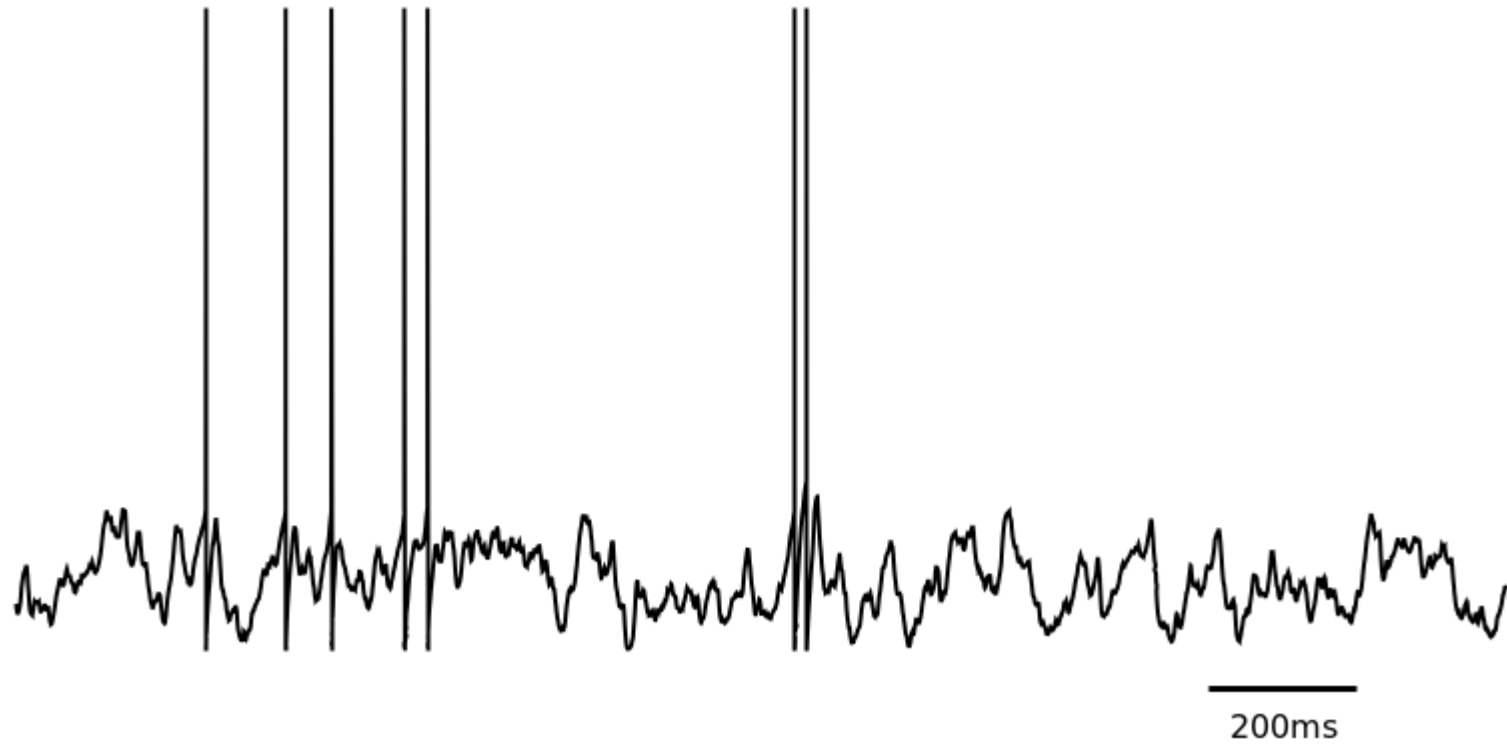
# Deep learning approach: Gradient descent on “smooth” cost function

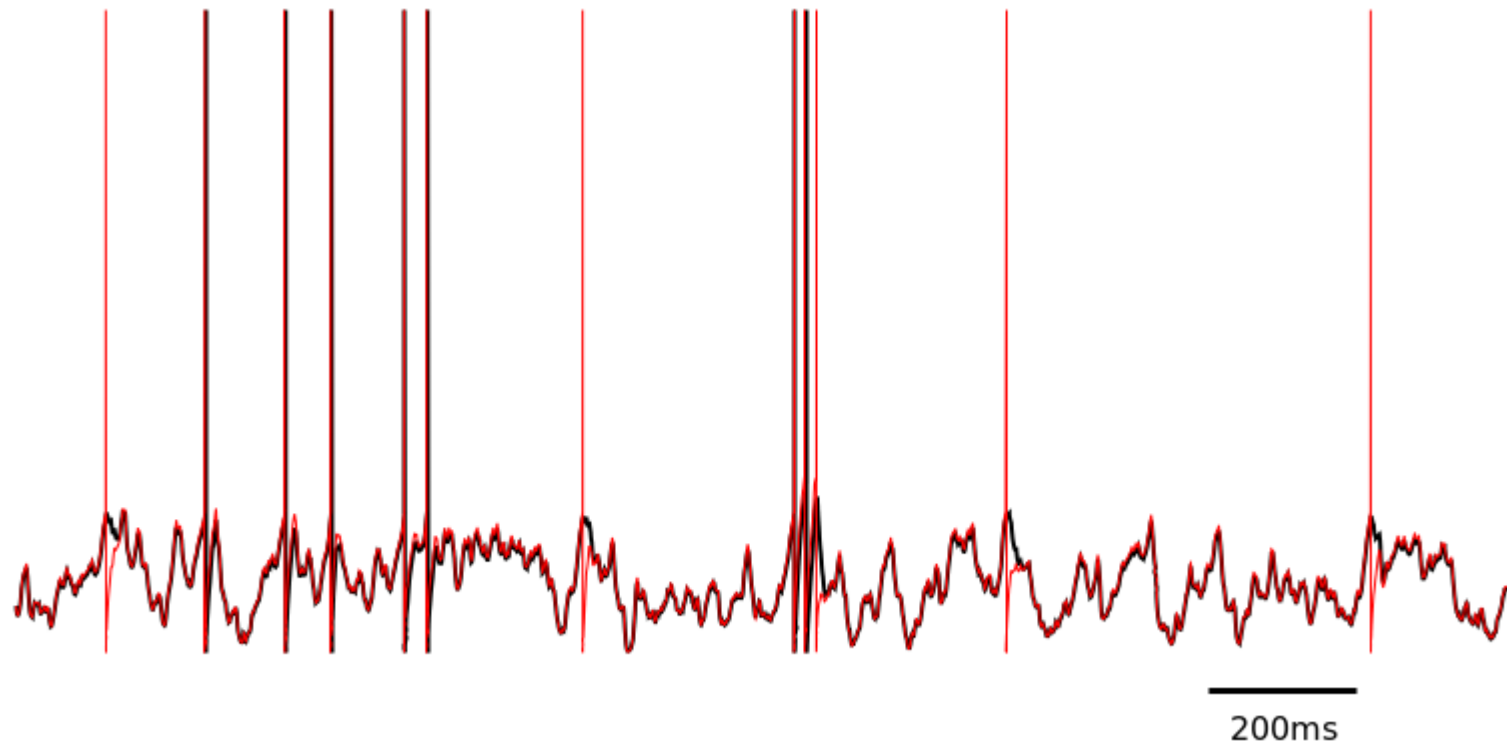
- Start with suitable cost function  $L$
- Take the derivative w.r.t. to synaptic weights
- Do gradient descent  $\frac{\partial w_k}{\partial t} \propto -\frac{\partial L}{\partial w_k}$

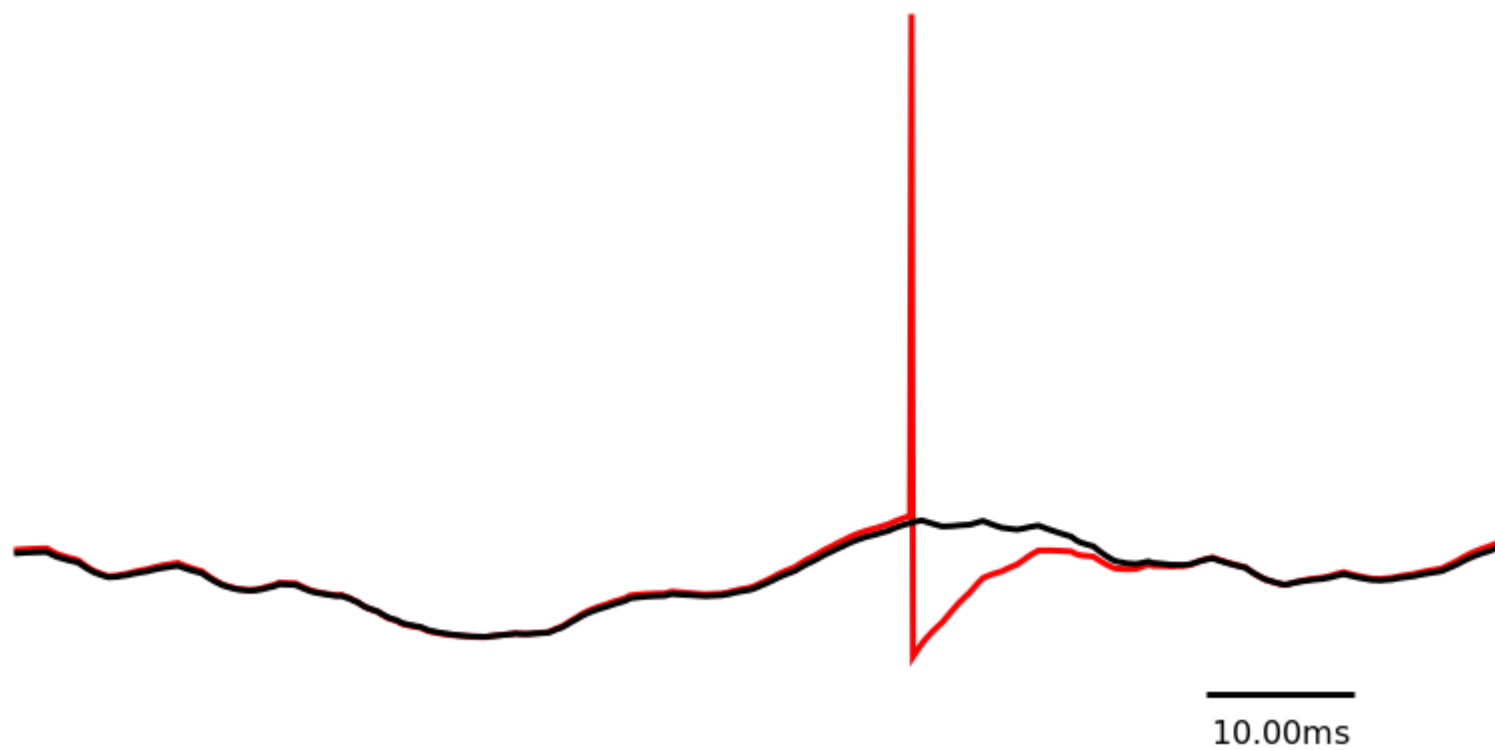


# Problems with gradient descent in spiking neural networks

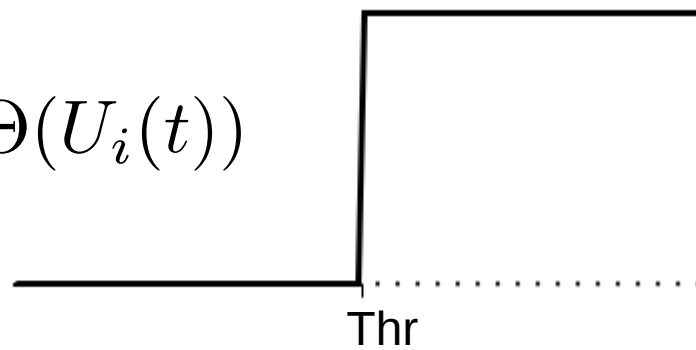
- Suitable cost function for spike trains
- Spikes are inherently non-differentiable
- Spiking neurons have history dependence due to spike reset
- Credit assignment in hidden layers





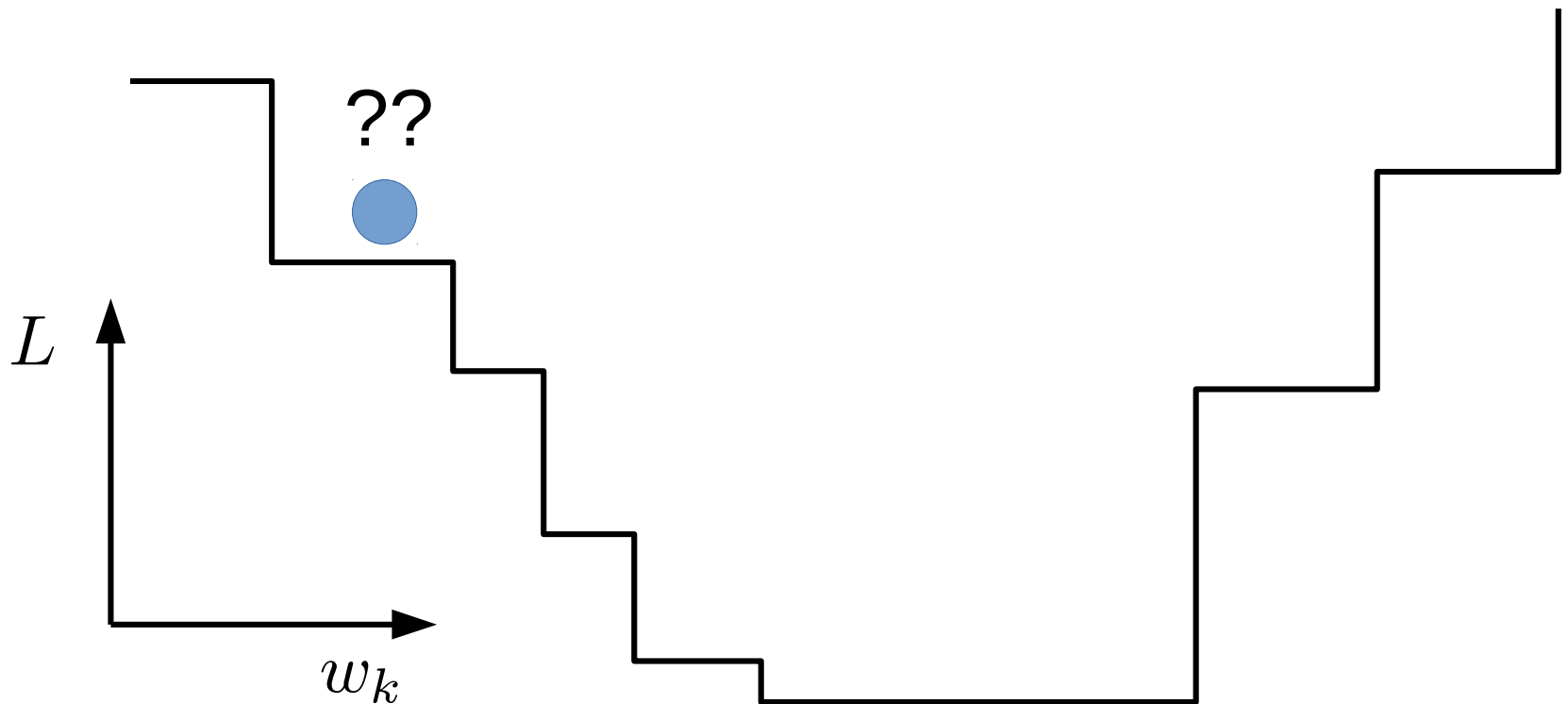


$$S(U_i(t)) \propto \Theta(U_i(t))$$

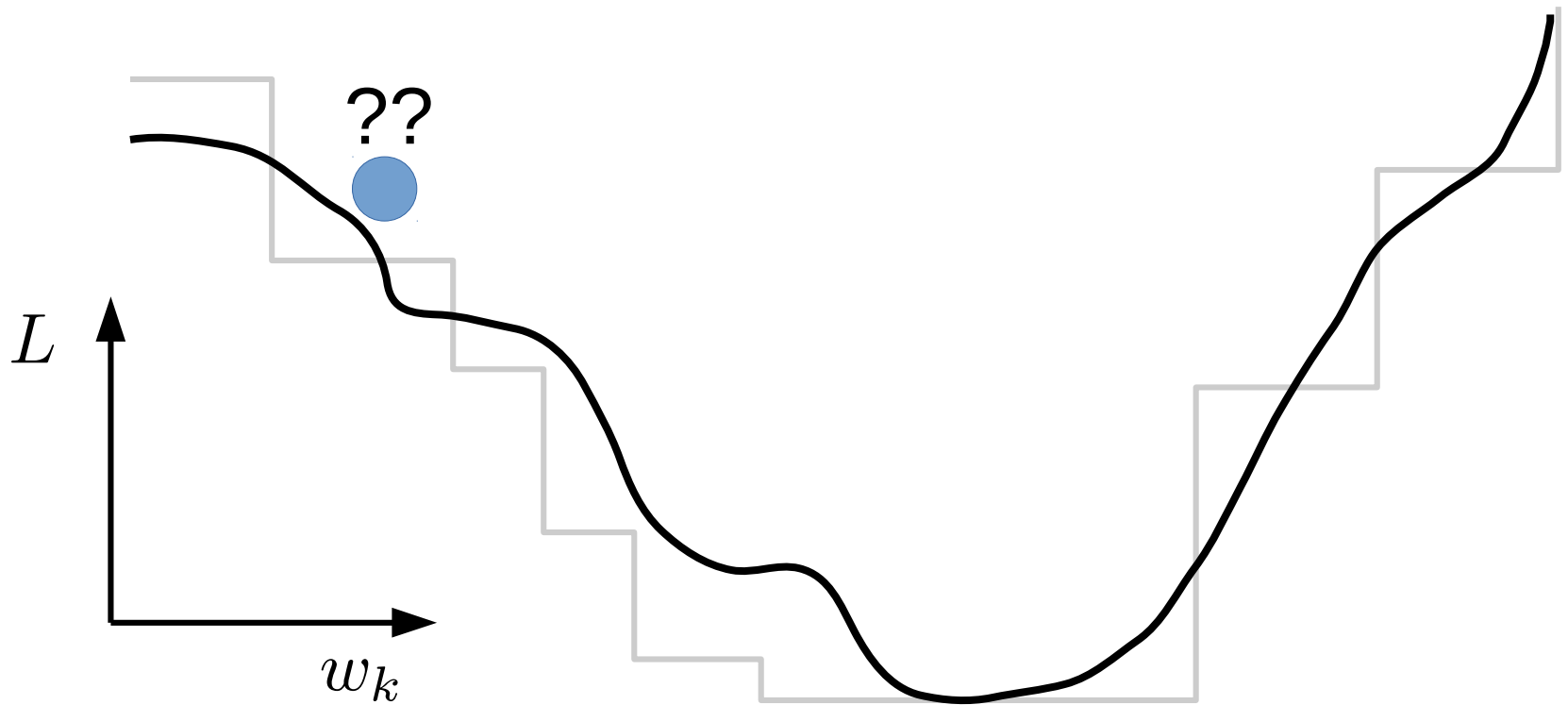


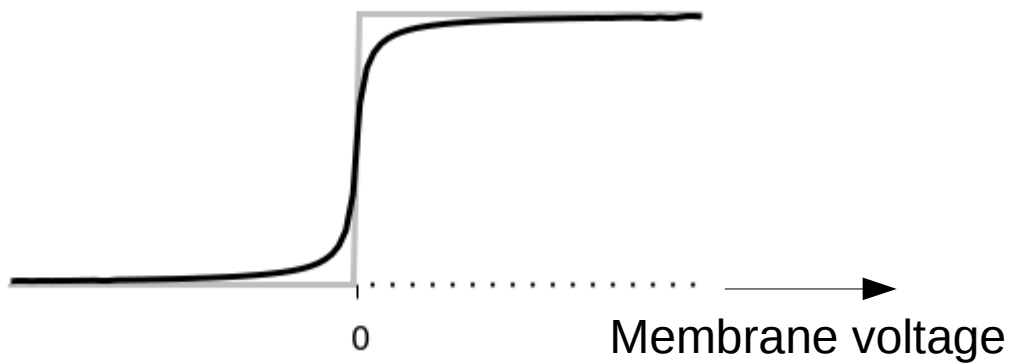
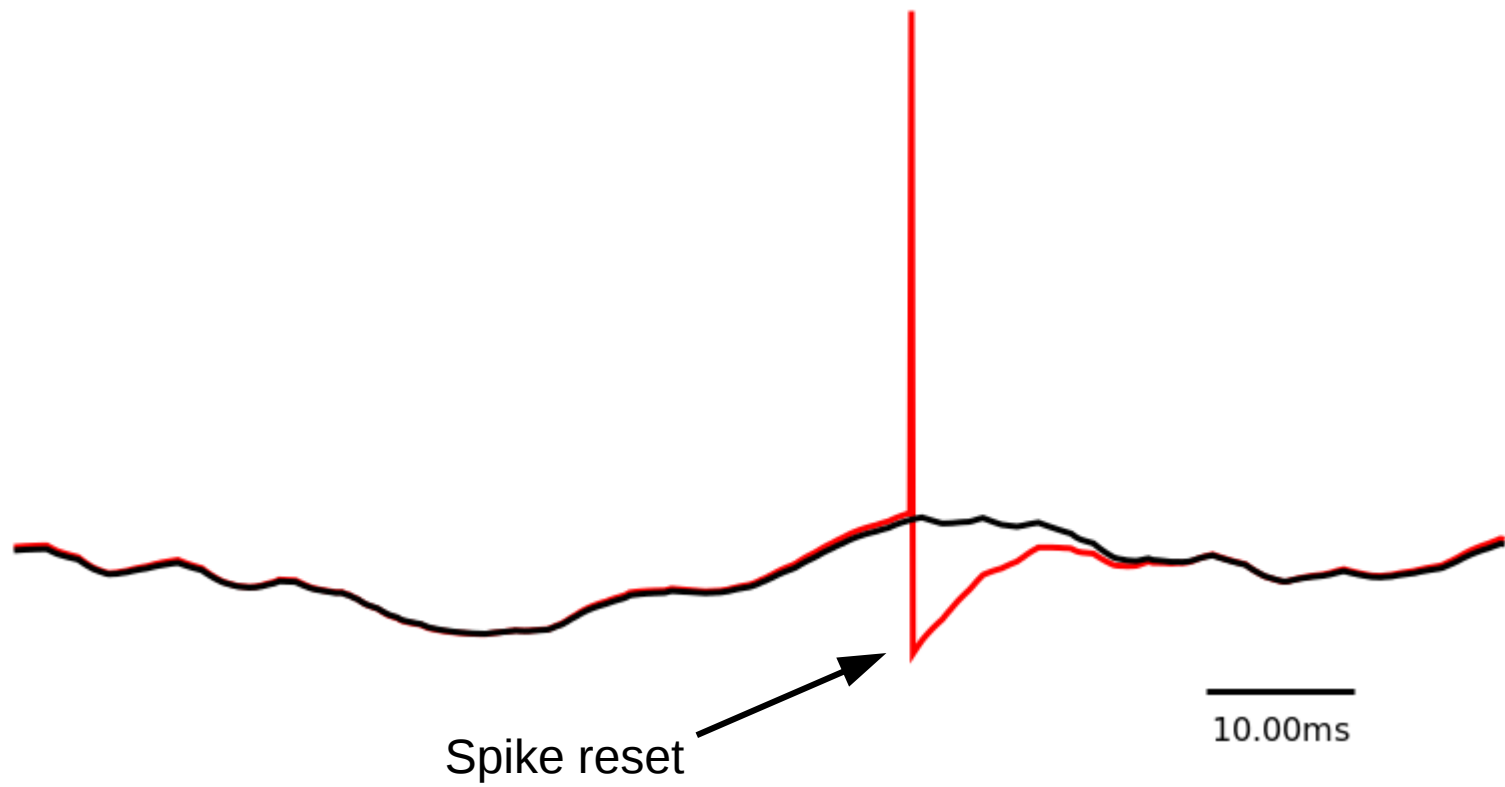


# Optimization landscape in spiking neural networks flat everywhere



# Idea: Smooth out the cost function



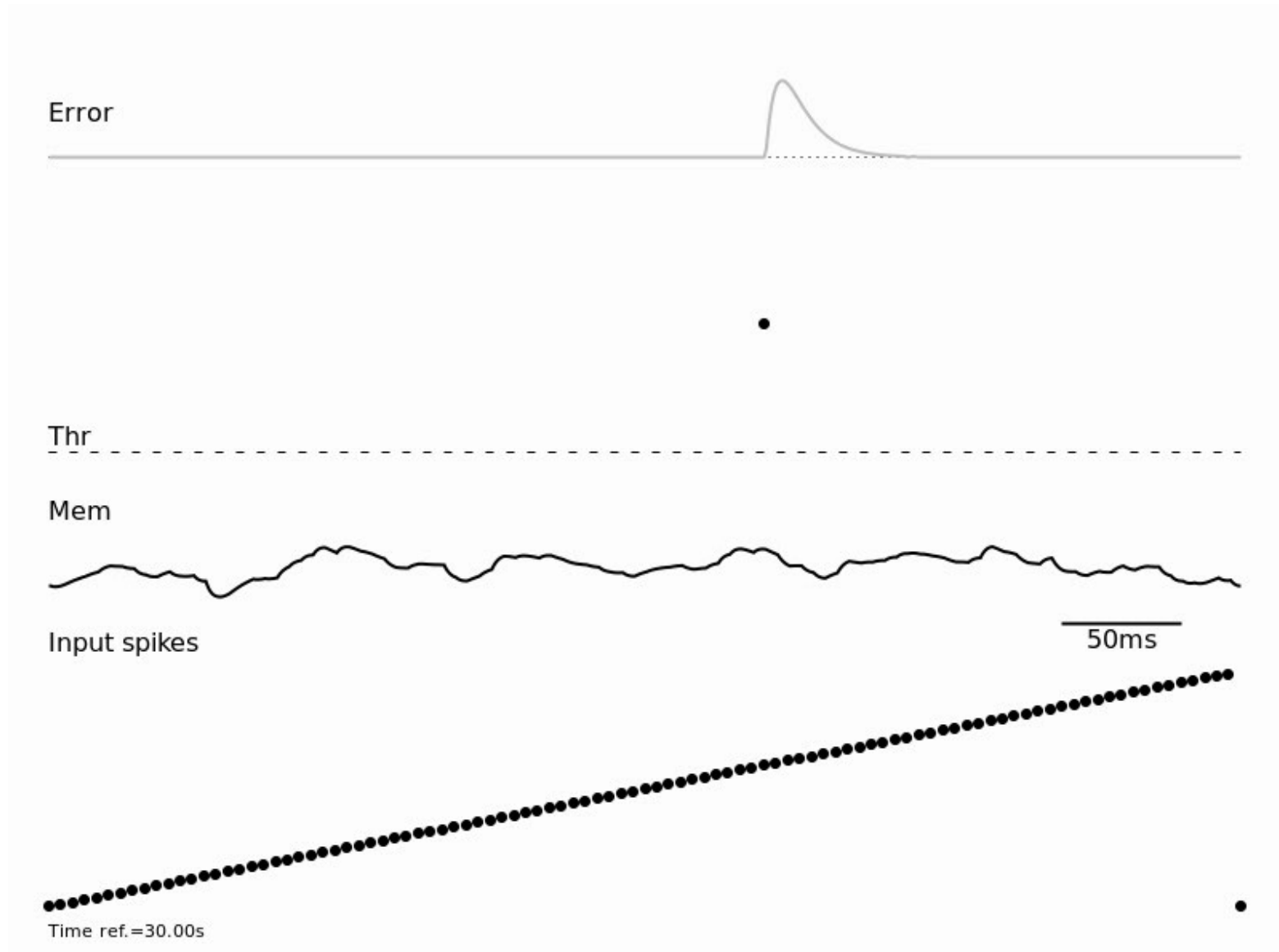


# The learning rule can be interpreted as a three factor rule

$$\frac{\partial w_{ij}}{\partial t} \equiv \underbrace{e_i(t)}_{\text{error signal}} \epsilon * \underbrace{(\epsilon * S_j(t))}_{\text{pre}} \underbrace{\sigma'(U_i))}_{\text{post}}$$

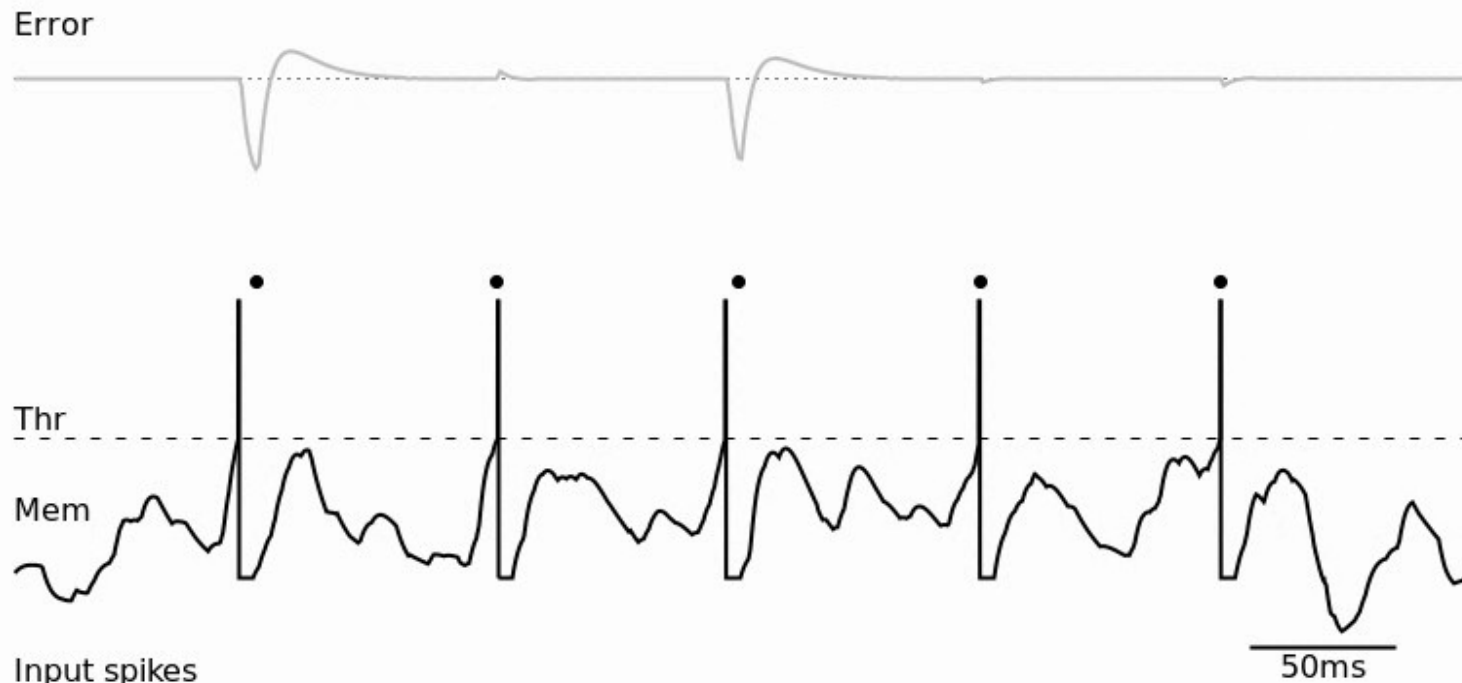
- Three factor rule
- Non-linear Hebbian
- Voltage-based
- Think of outer convolution as eligibility trace

# Sequential inputs → single spike

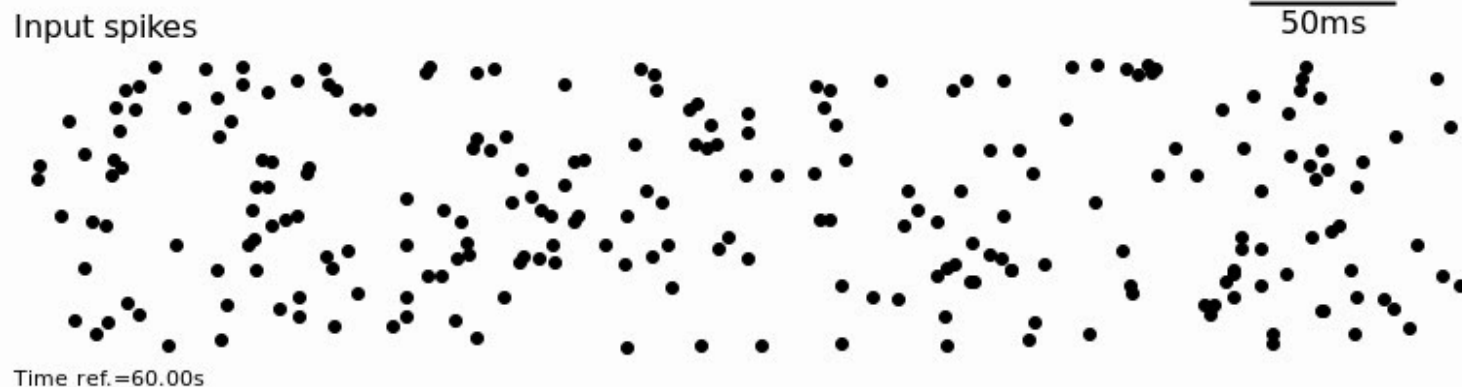


# Learning of output spike train

Output:



Input:

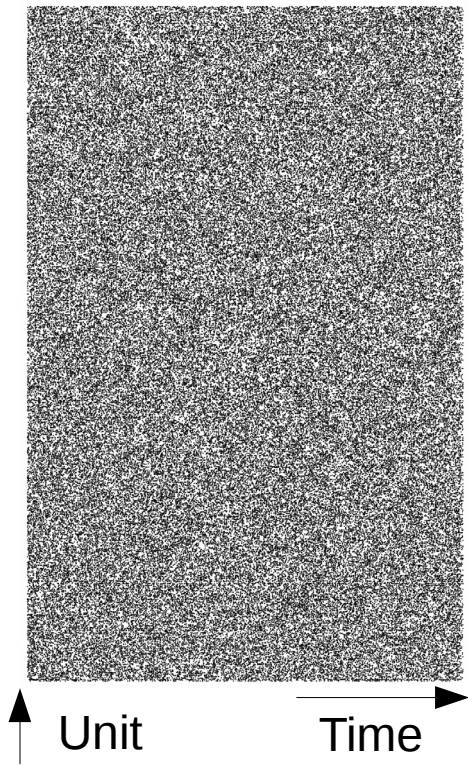


Time ref.=60.00s

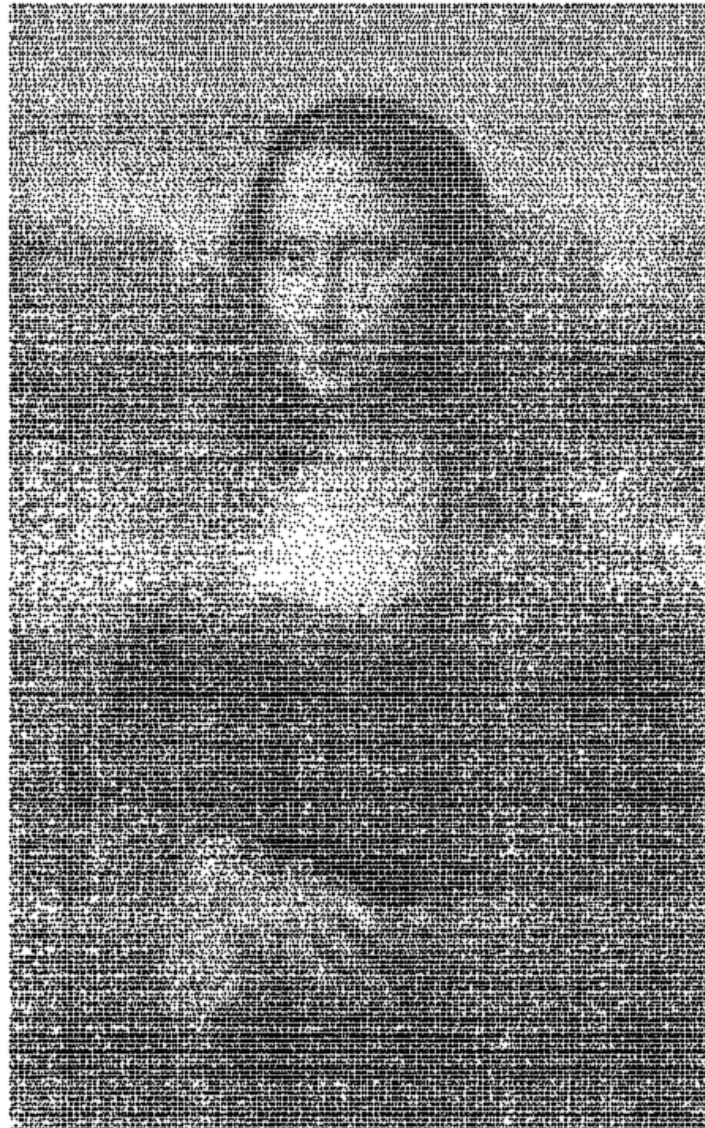


# Training many outputs in parallel

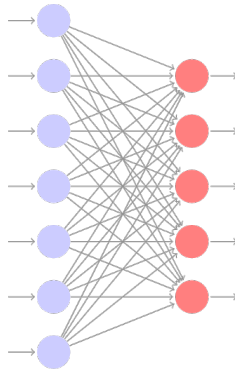
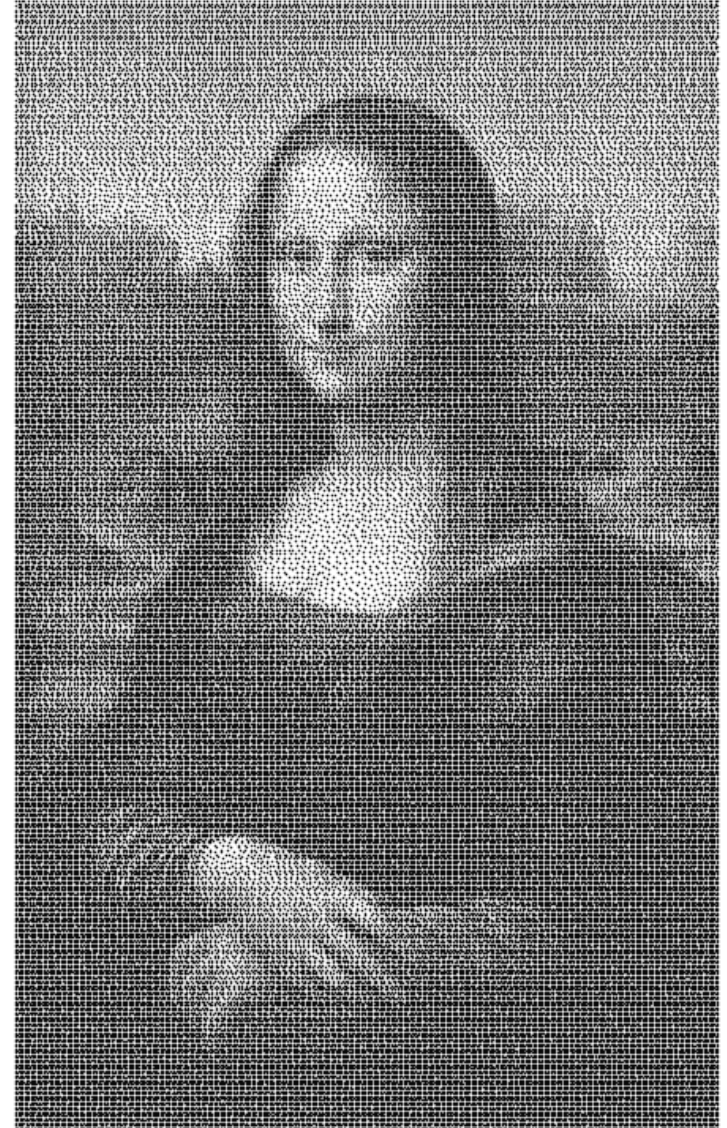
1000 inputs



500 output neurons



Target spike trains



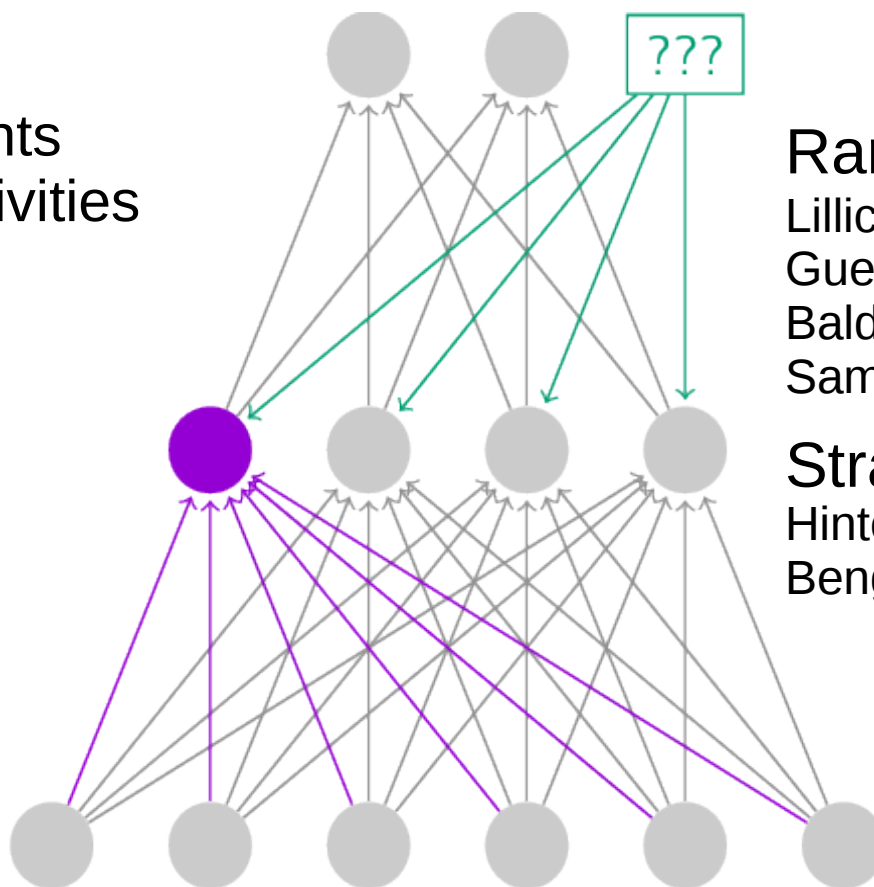


# Can we take this to multiple layers?

$$\frac{\partial w_{ij}}{\partial t} \equiv \sum_k e_k(t) \epsilon * [w_{ki} \epsilon * (\epsilon * S_j(t) \sigma'(U_i)) \sigma'(U_k)]$$

## Problems:

- Symmetric weights
- Downstream activities



## Random feedback

Lillicrap et al. (2014, 2016)

Guergiuev et al. (2016)

Baldi et al. (2016)

Samadi et al. (2017)

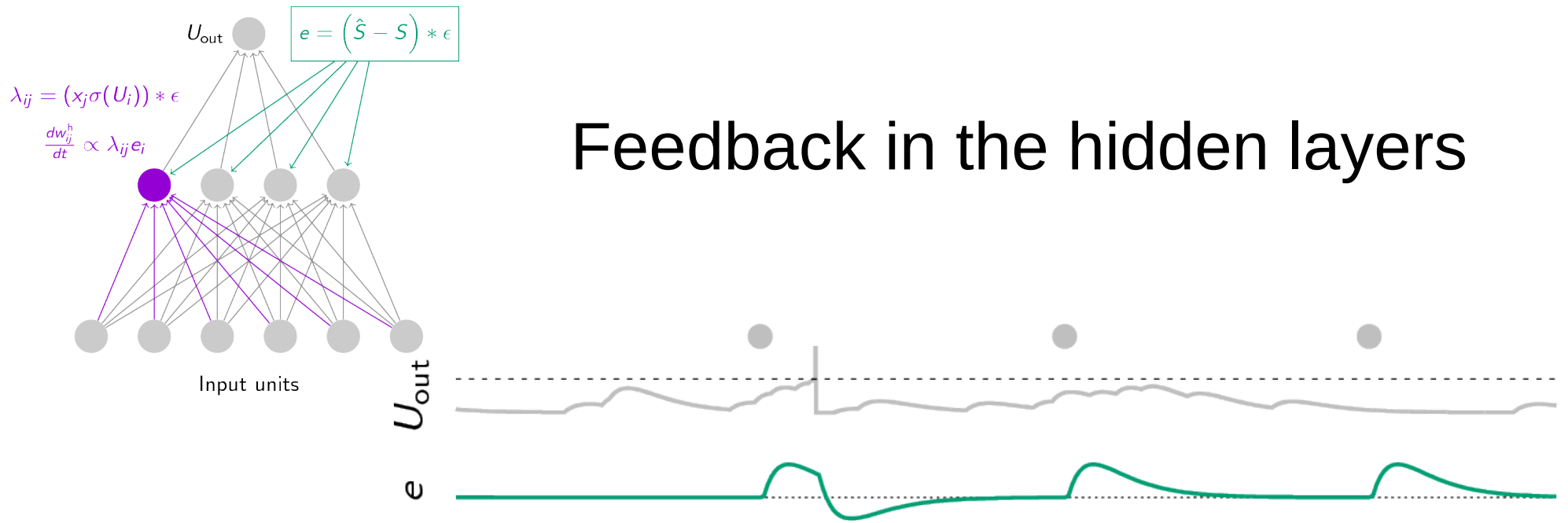
## Straight-through estim.

Hinton (2012)

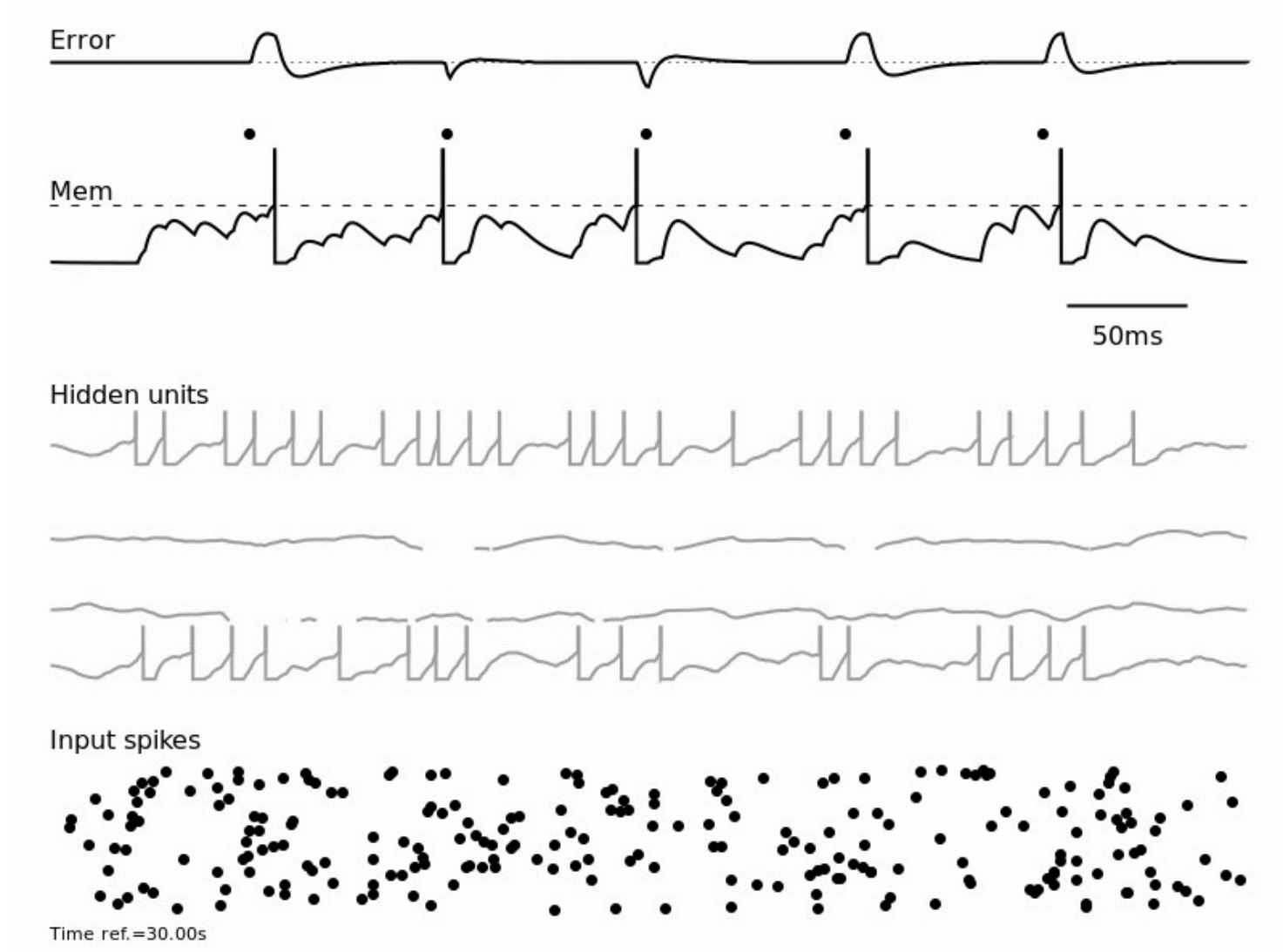
Bengio et al. (2013)



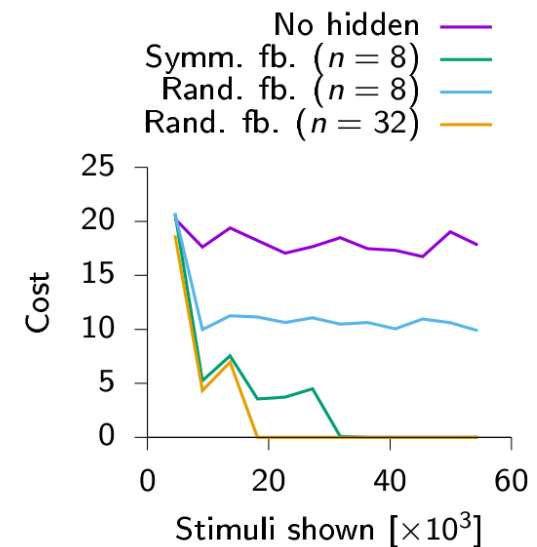
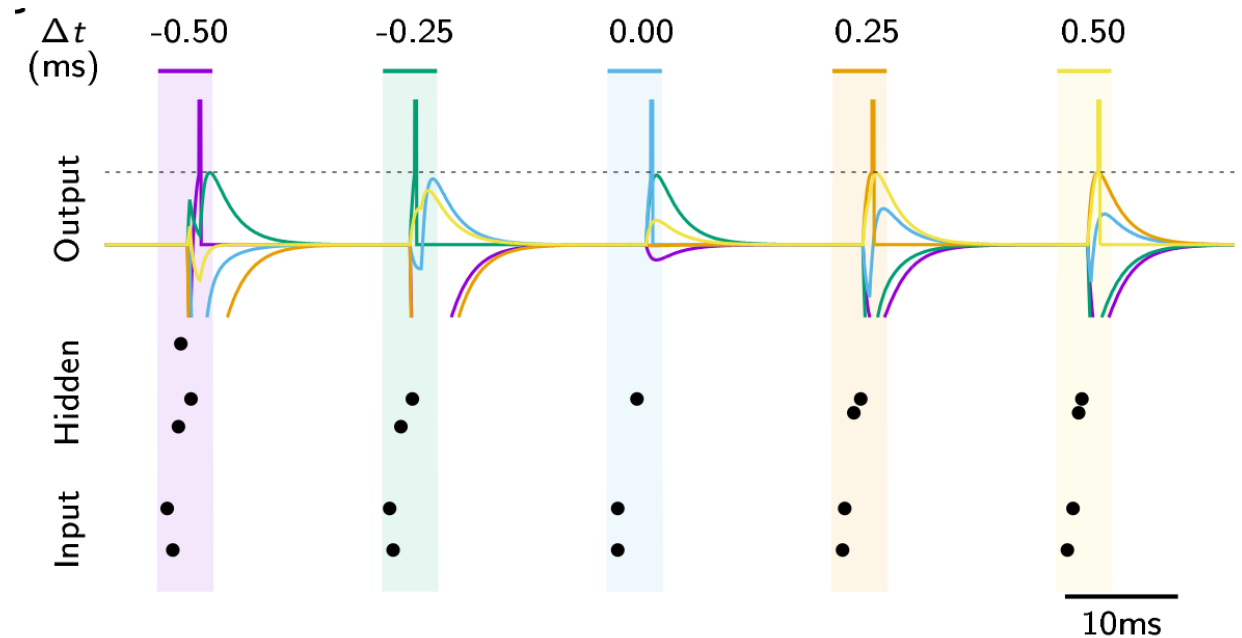
# Feedback in the hidden layers



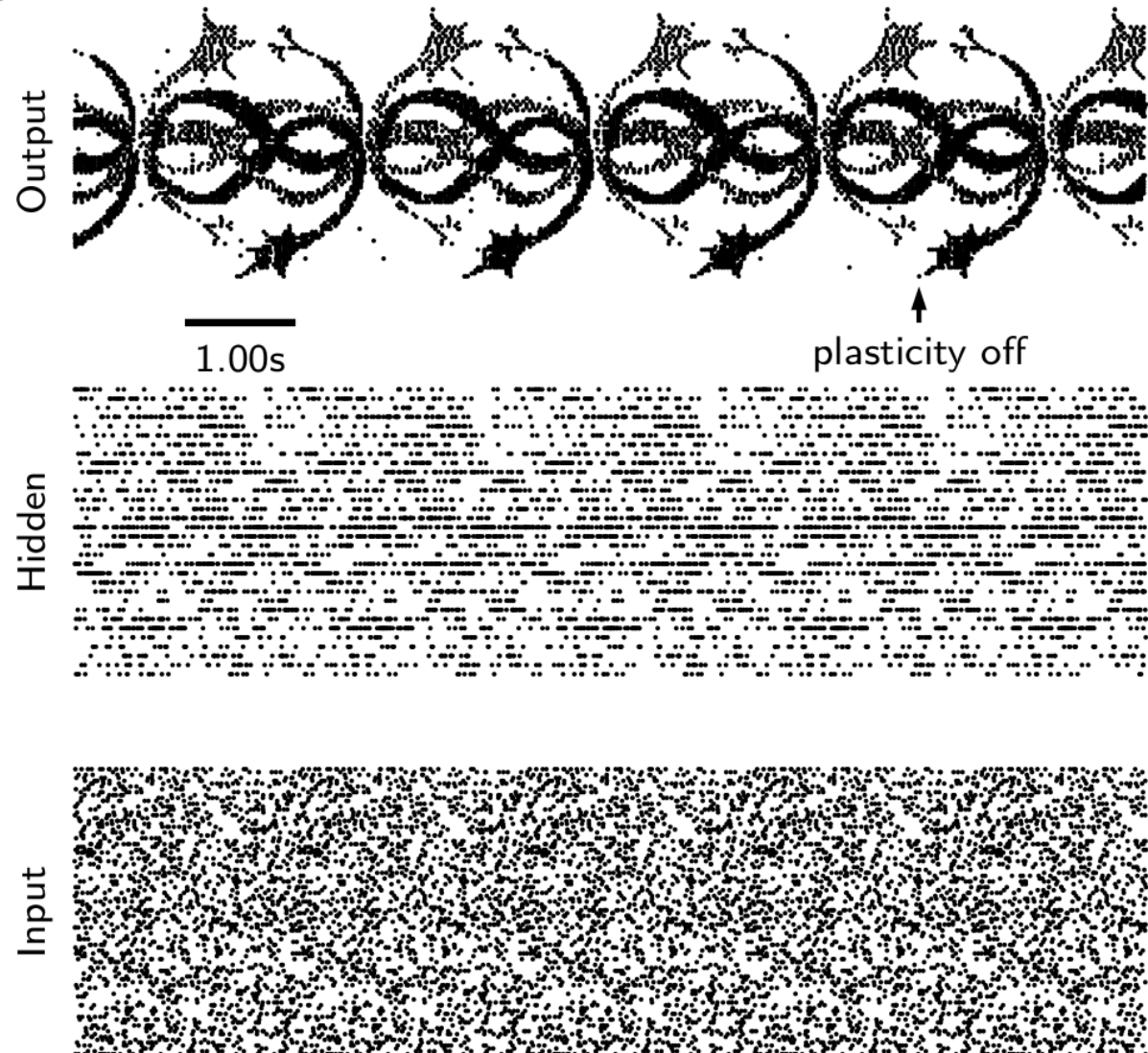
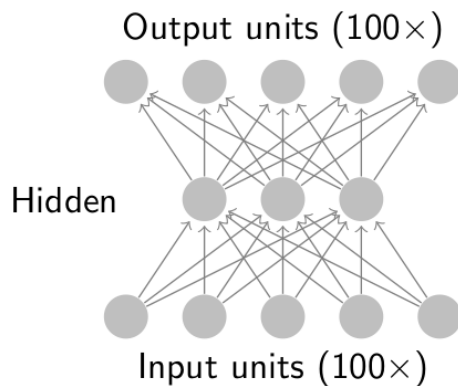
# Random feedback, one hidden layer



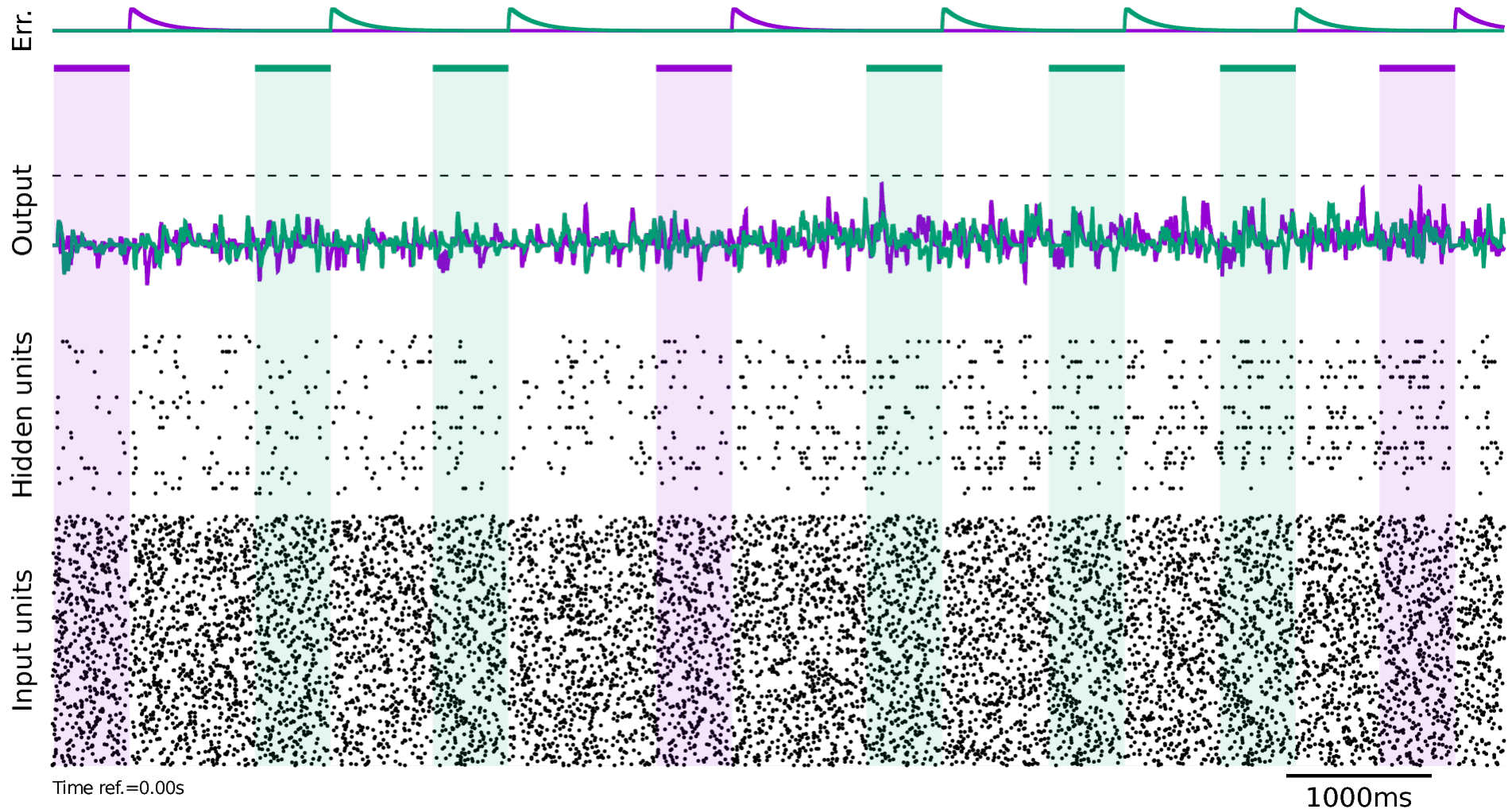
# Non-linearly separable problems



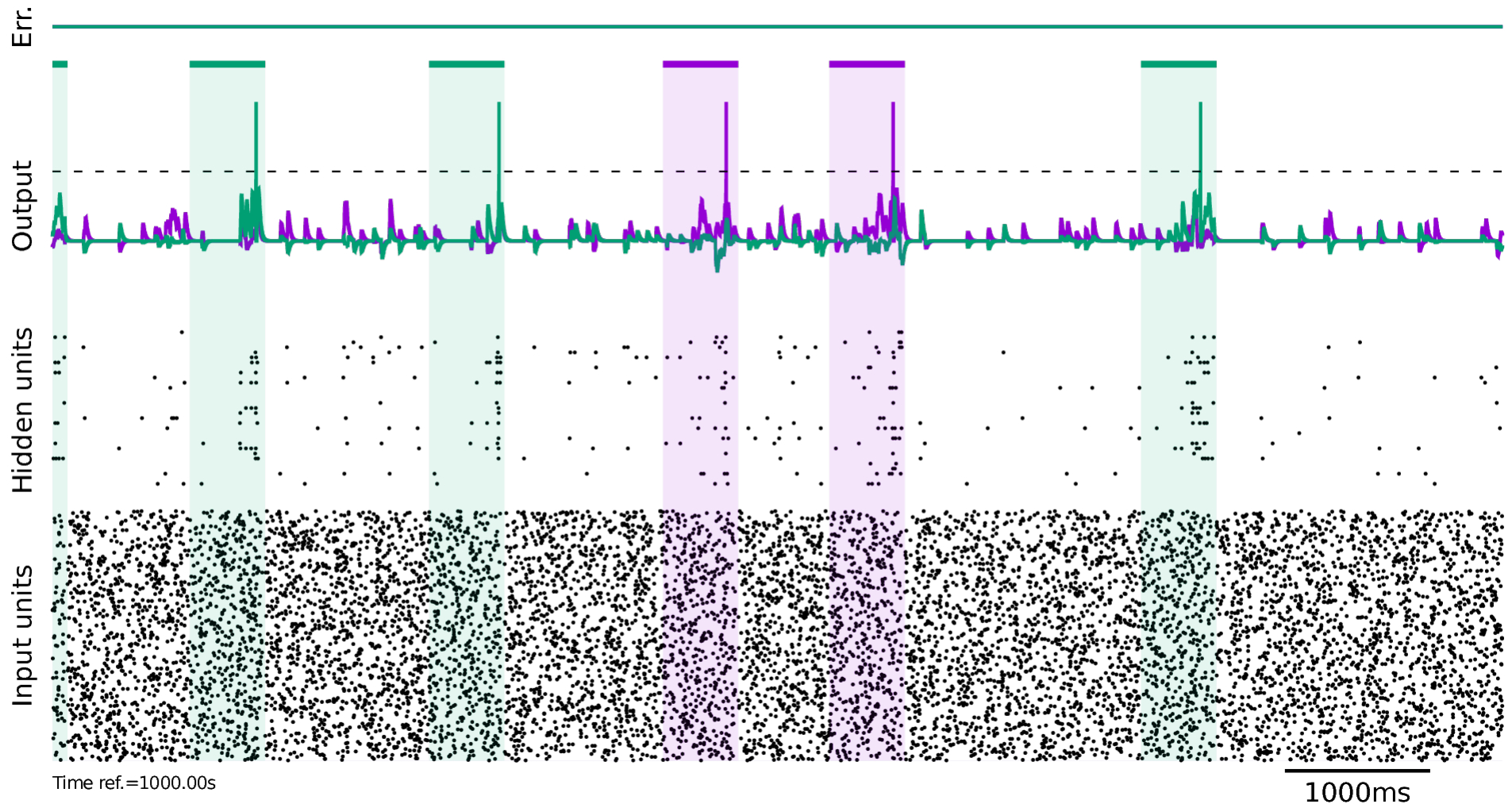
# Learning complex spatiotemporal sequences



# Classifying temporally coded input patterns

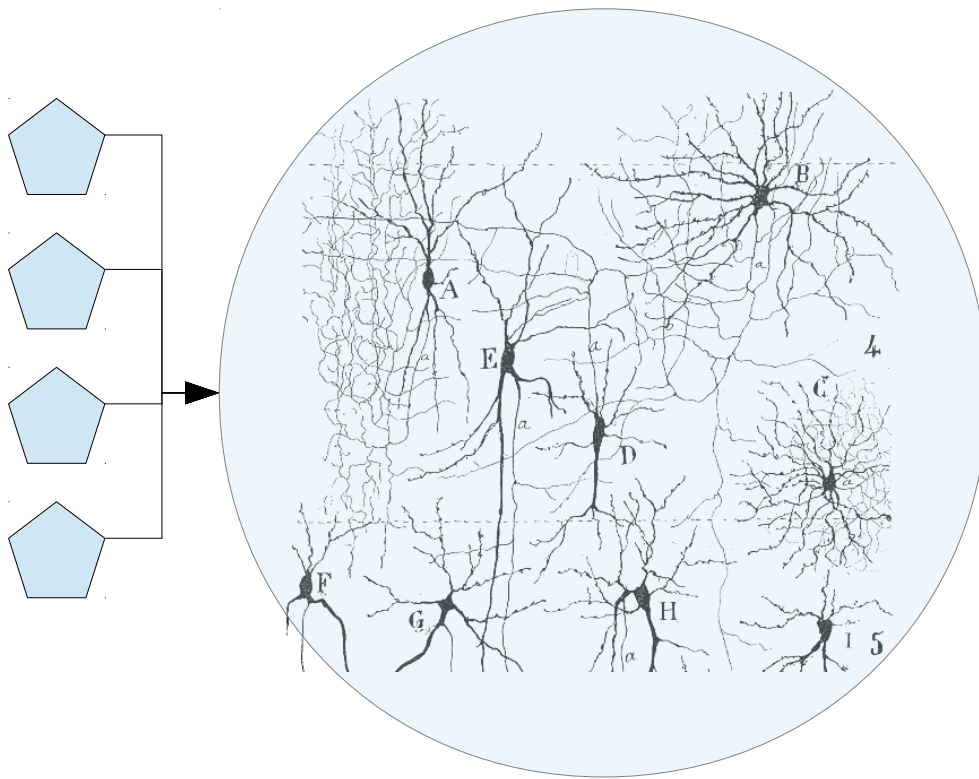


# Classifying temporally coded input patterns

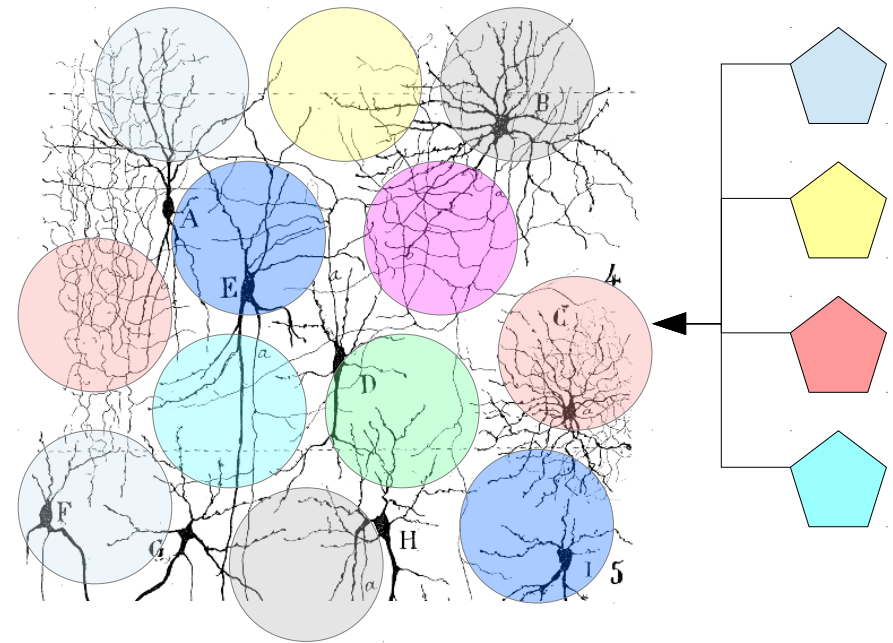




# Heterogeneous feedback is important



Global third factor



Heterogeneous third factor



# Summary

- Started from cost function approach
- Method to teach spiking nets to solve non-trivial temporally coded problems
- Learning rule has a simple interpretation in a biological context

# Acknowledgements

- Part 1: Everton Agnes & Wulfram Gerstner
- Part 2: Surya Ganguli & Gang

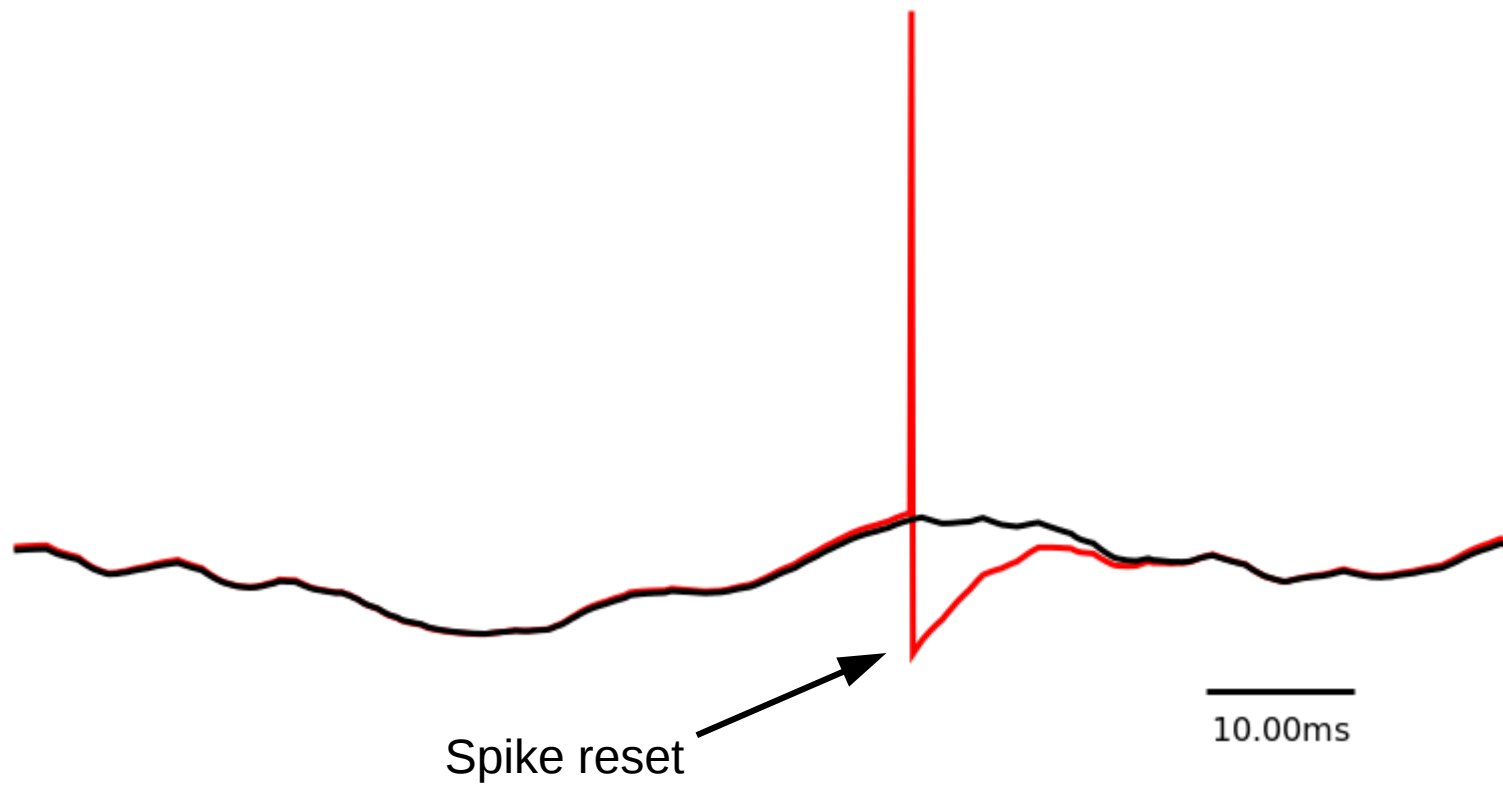


SWISS NATIONAL SCIENCE FOUNDATION

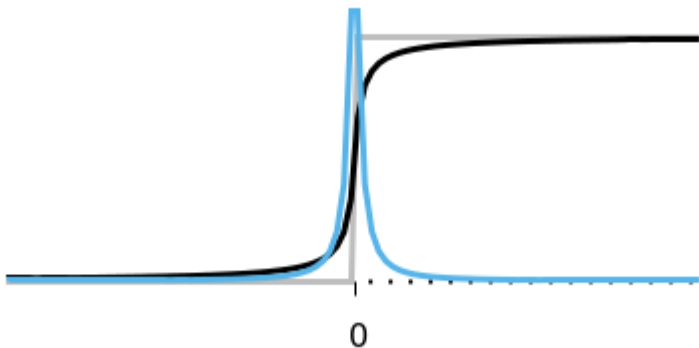
F. Zenke - [fzenke.net](http://fzenke.net)







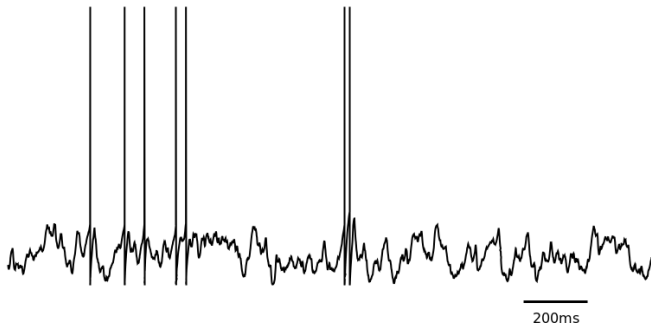
$$S(U_i(t)) \propto \Theta(U_i(t)) \approx \sigma(U_i(t))$$



$$\frac{\partial S_i}{\partial w_{ij}} \approx \sigma'(U_i) \frac{\partial U_i}{\partial w_{ij}}$$

need  $\frac{\partial U_i}{\partial w_{ij}}$

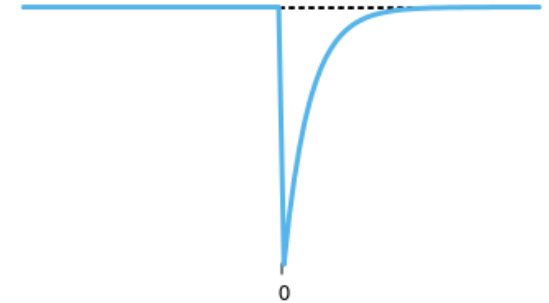
Membrane voltage



PSP kernel  $\epsilon$



Reset kernel  $\eta$



- Leaky integrate-and-fire neuron model
- Dynamics defined by ODE
- Equivalent to the spike-response-model (SRM0)
- Can be written with temporal convolutions

$$U_i(t) = \sum_j w_{ij} \epsilon * S_j(t) + \cancel{\eta * S_i(t)}$$

$$\frac{\partial S_i}{\partial w_{ij}} \approx \underbrace{\sigma'(U_i)}_{\text{post}} \underbrace{\epsilon * S_j(t)}_{\text{pre}}$$

# The learning rule can be interpreted as a three factor rule

$$-\frac{\partial L}{\partial w_k} = \int_{-\infty}^t \underbrace{\epsilon * (\hat{S}_i(s) - S_i(s))}_{\equiv e_i(t)} \epsilon * \frac{\partial S_i(t)}{\partial w_k} ds$$

$$\frac{\partial w_{ij}}{\partial t} \equiv \underbrace{e_i(t)}_{\text{error signal}} \underbrace{\epsilon * S_j(t)}_{\text{pre}} \underbrace{\sigma'(U_i)}_{\text{post}}$$

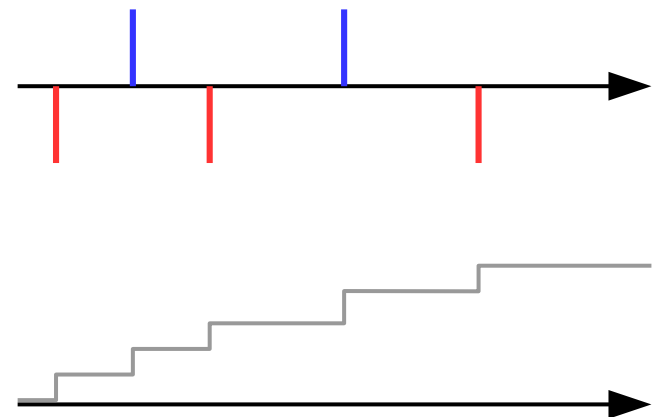
- Three factor rule
- Non-linear Hebbian
- Voltage-based
- Think of outer convolution as eligibility trace

# Which spike train metric to use

Spike train:

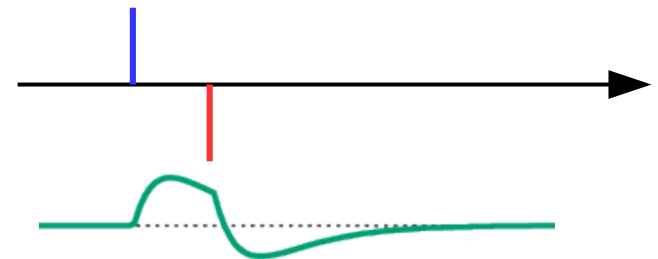
$$S_i(t) = \sum_k \delta(t - t_i^k)$$

$$L_{\text{naive}} = \int_{-\infty}^t \left| \hat{S}_i(s) - S_i(s) \right| ds$$



van Rossum distance:

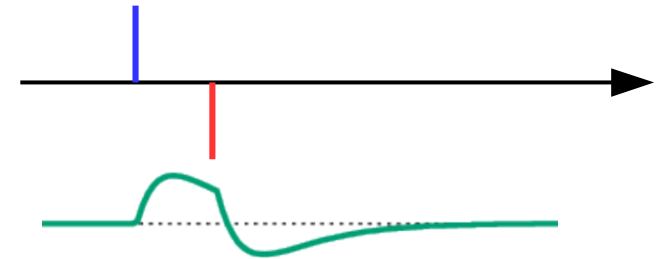
$$L = \int_{-\infty}^t \left( \epsilon * \hat{S}_i(s) - \epsilon * S_i(s) \right)^2 ds$$





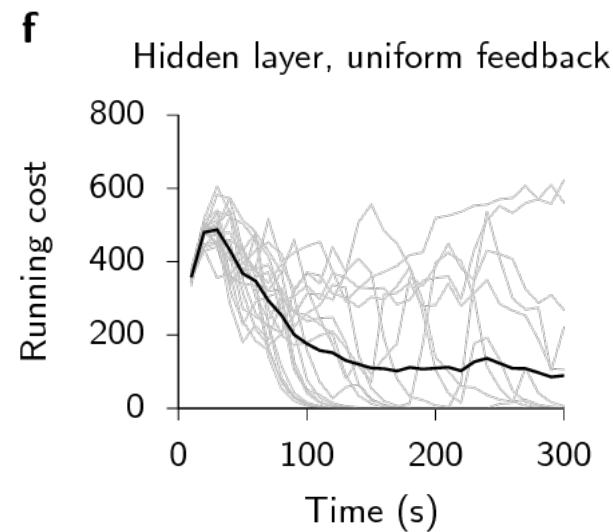
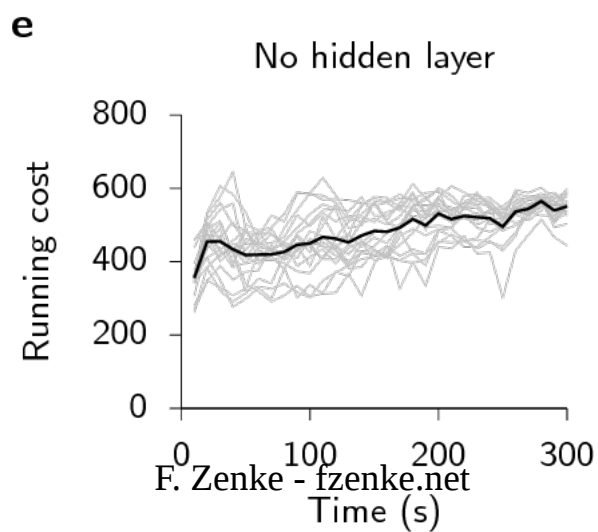
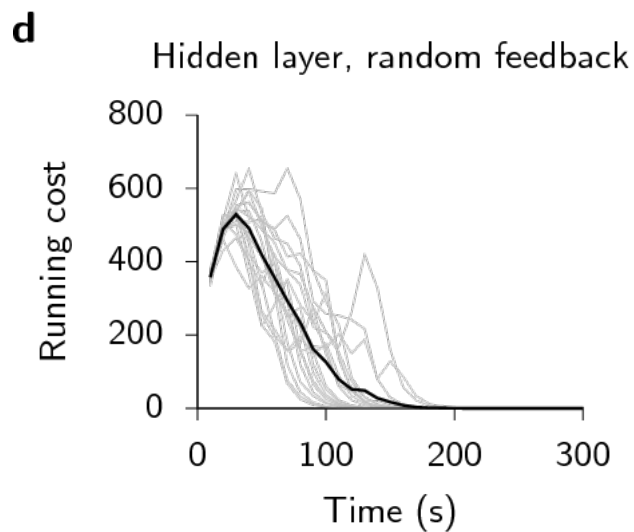
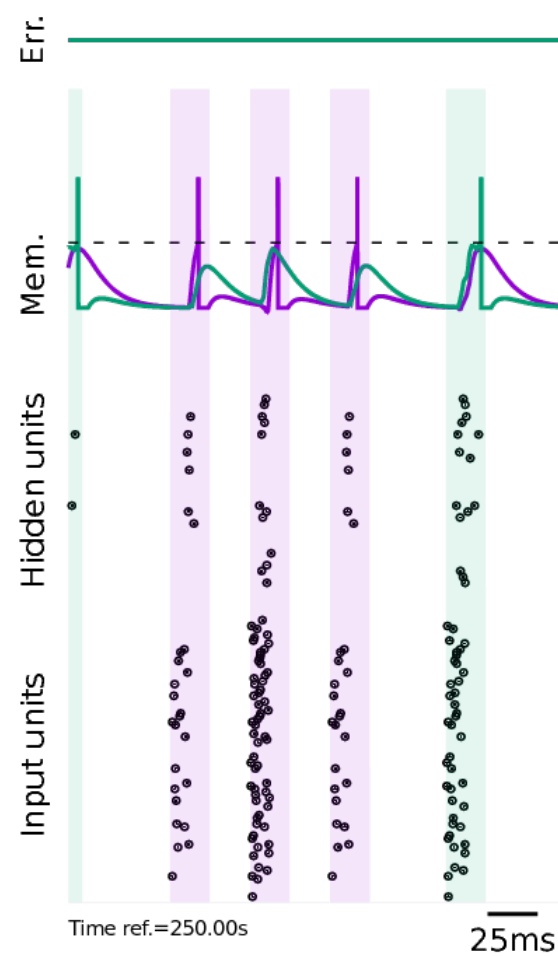
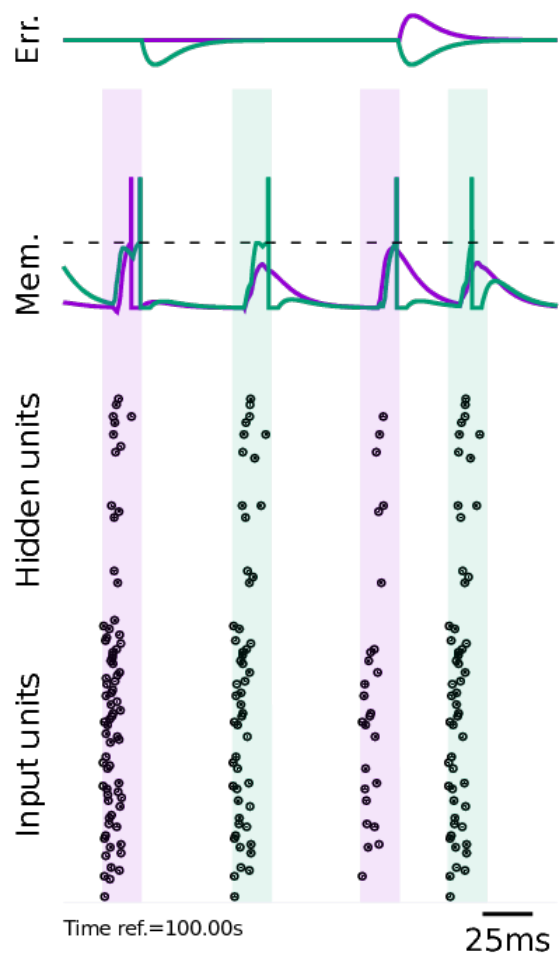
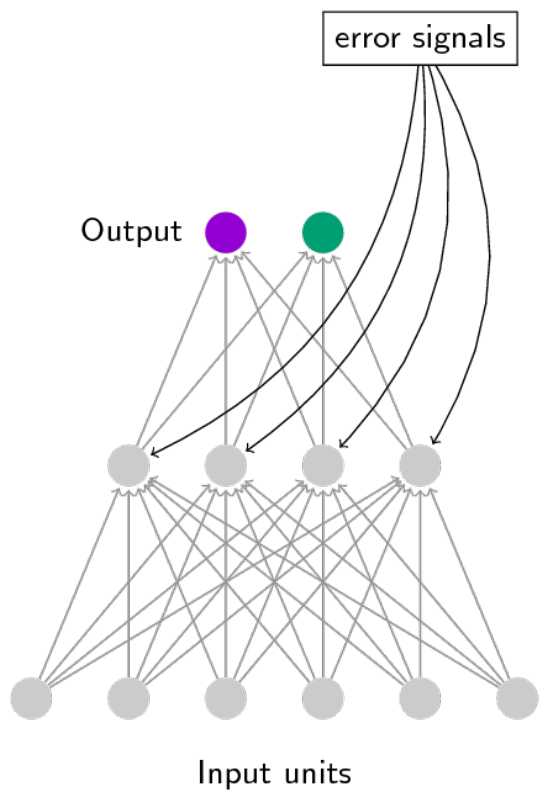
# Derivative of a spike train is zero almost everywhere

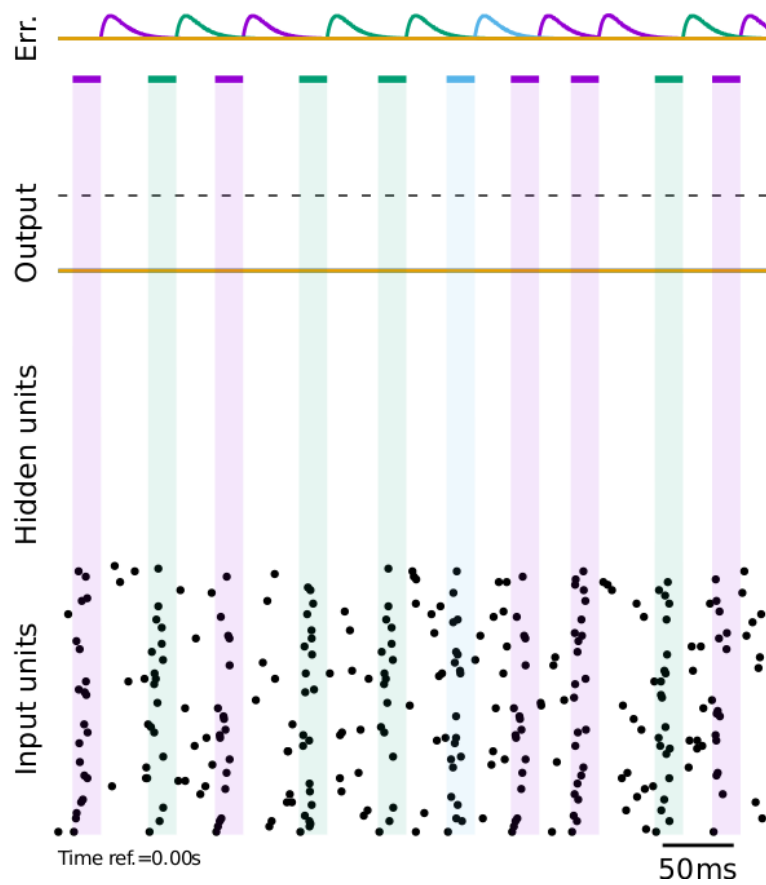
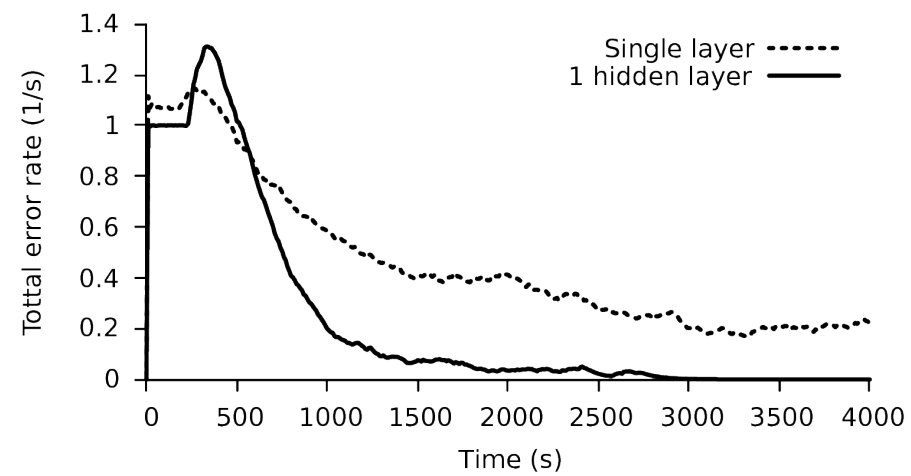
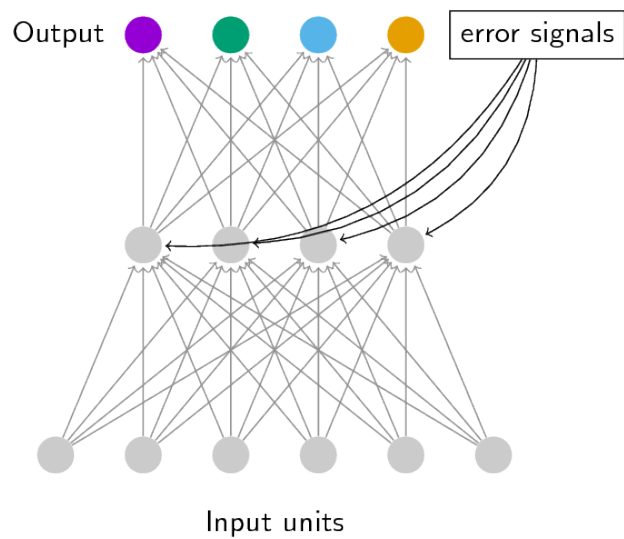
$$L = \frac{1}{2} \int_{-\infty}^t \left( \epsilon * \left( \hat{S}_i(s) - S_i(s) \right) \right)^2 ds$$

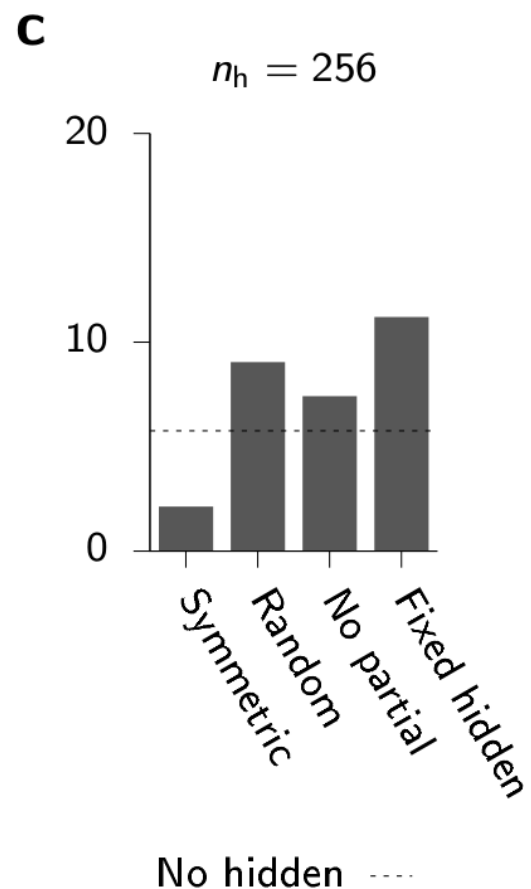
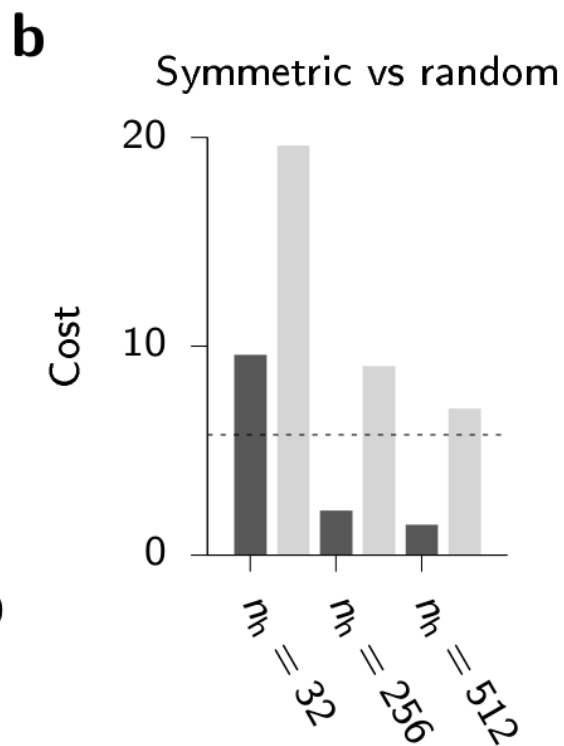
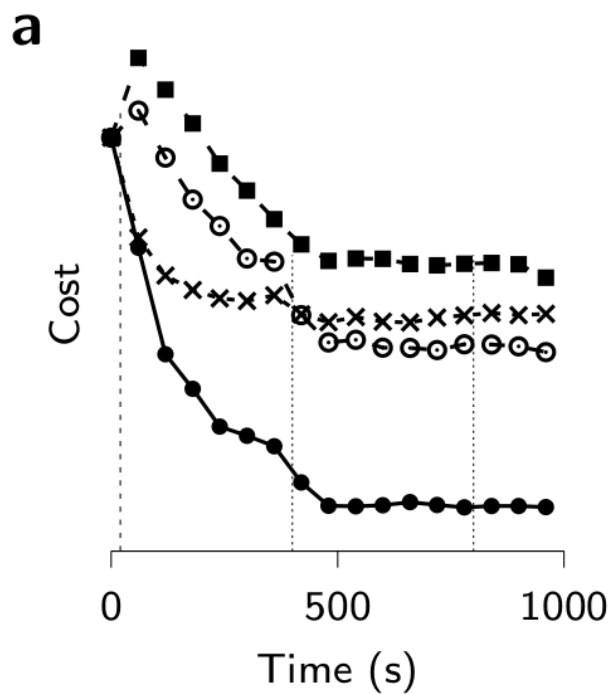


D'Oh!

$$-\frac{\partial L}{\partial w_k} = \int_{-\infty}^t \epsilon * \left( \hat{S}_i(s) - S_i(s) \right) \epsilon * \frac{\partial S_i(t)}{\partial w_k} ds$$







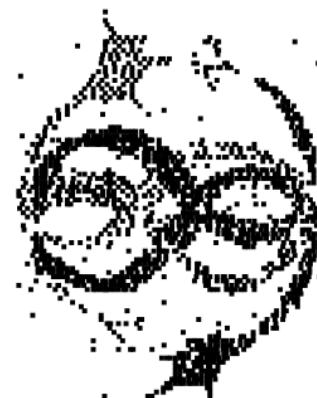
**d** Symm.,  $n_h = 256$



**e** Random (unreg.)



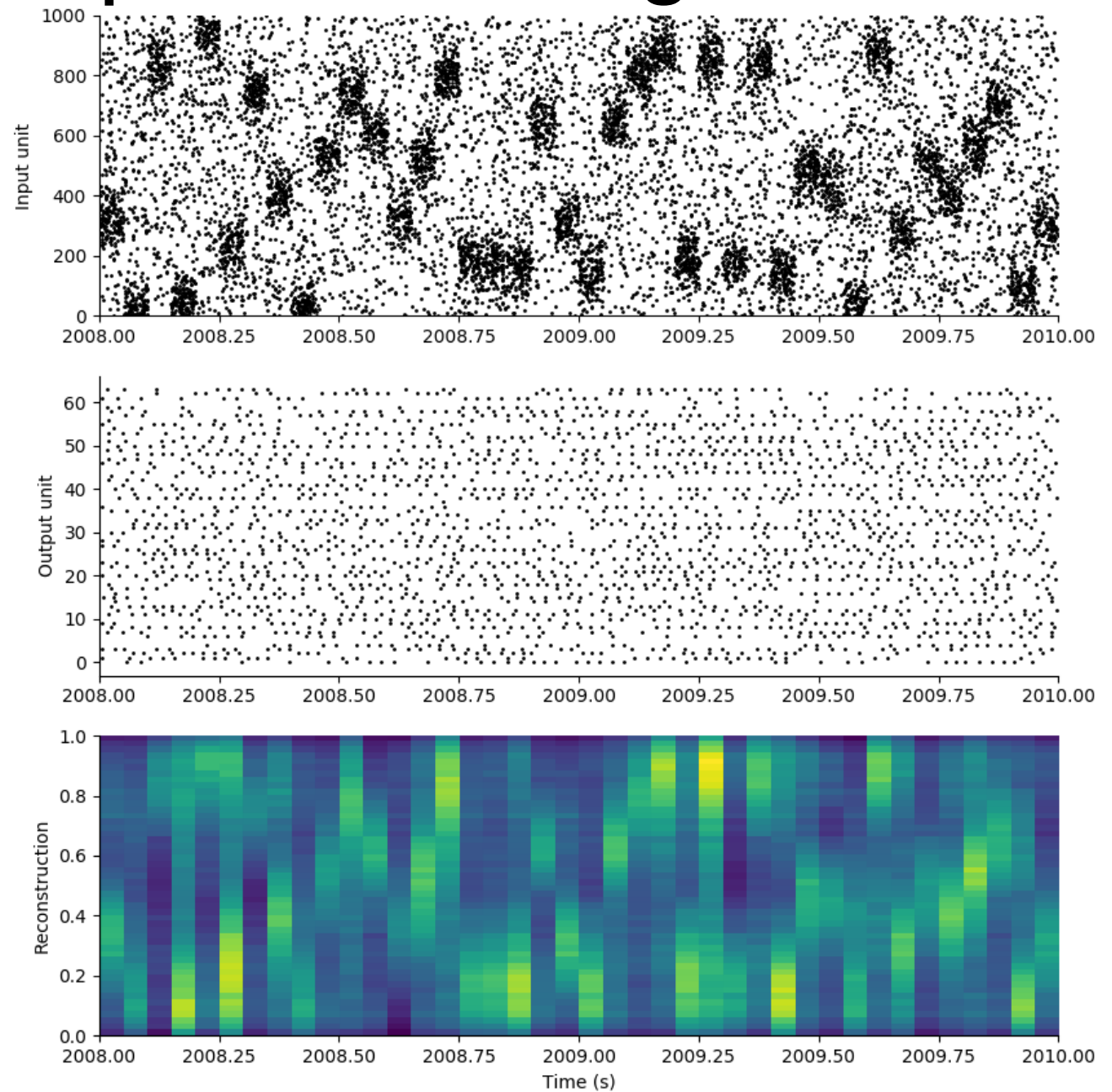
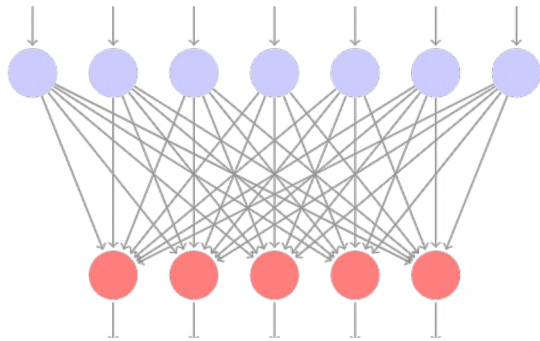
**f** Random



**g** No partial

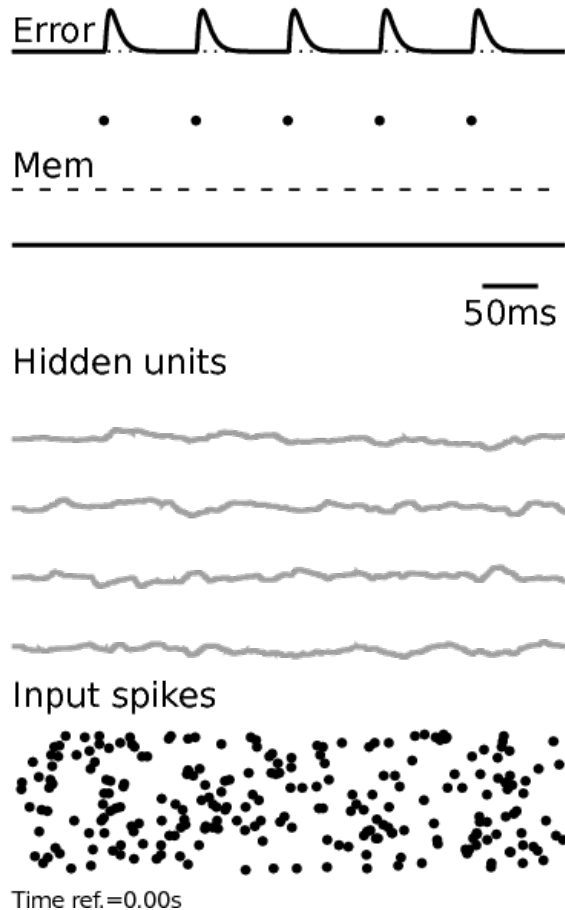


# Subspace learning

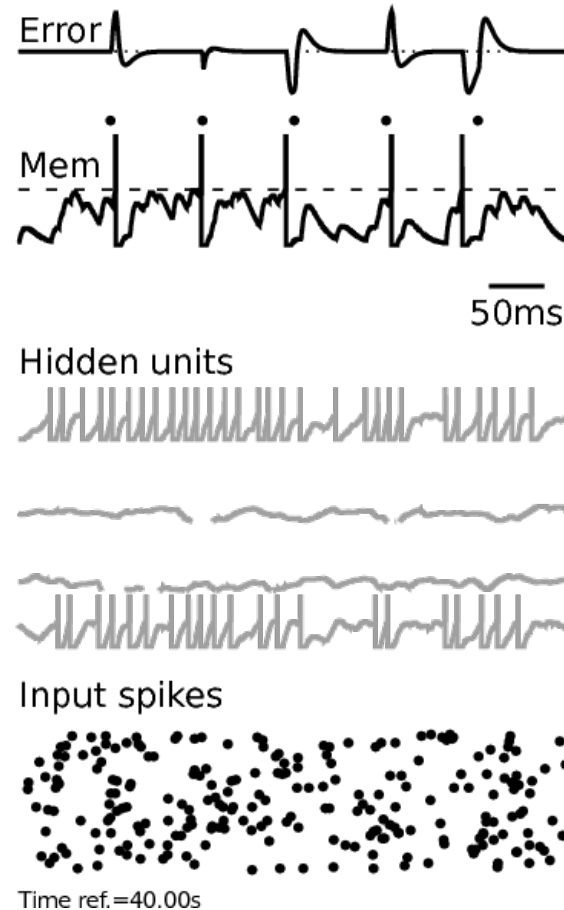


# Random feedback, one hidden layer

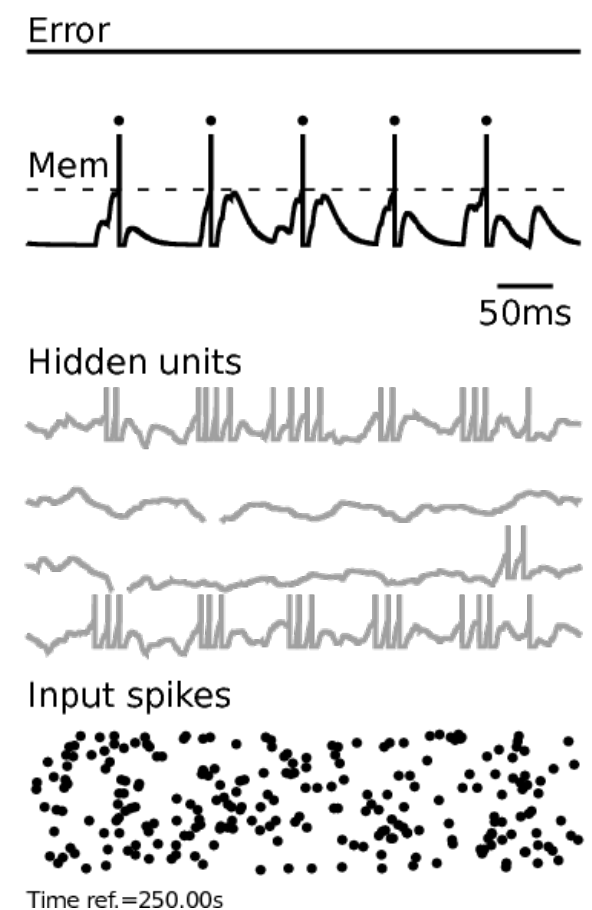
d



e

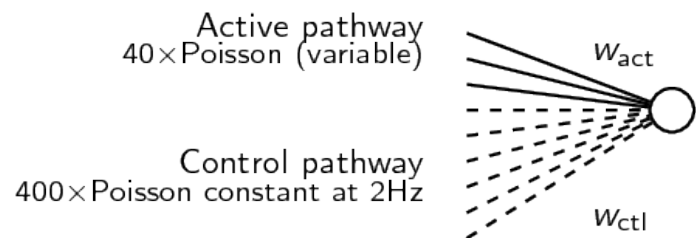
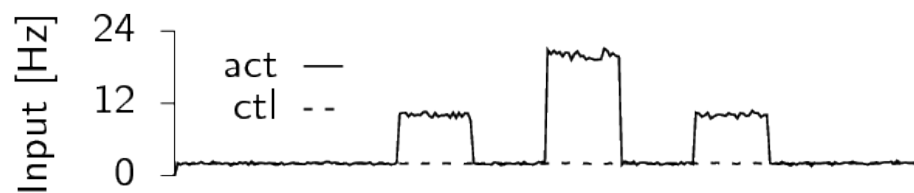
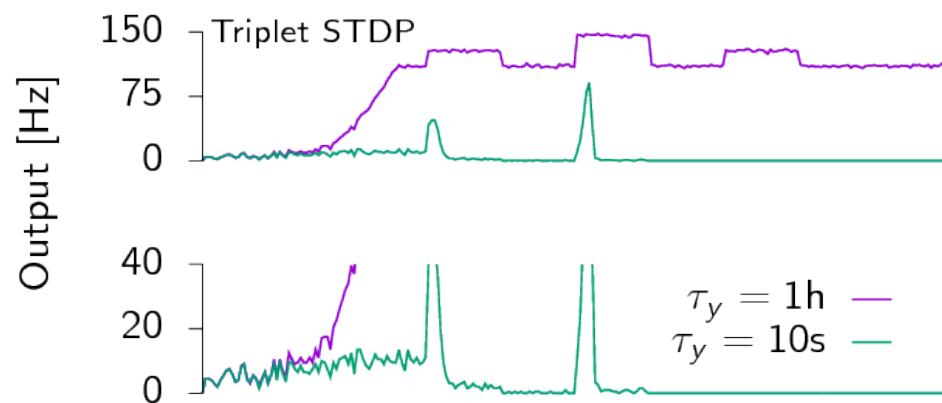
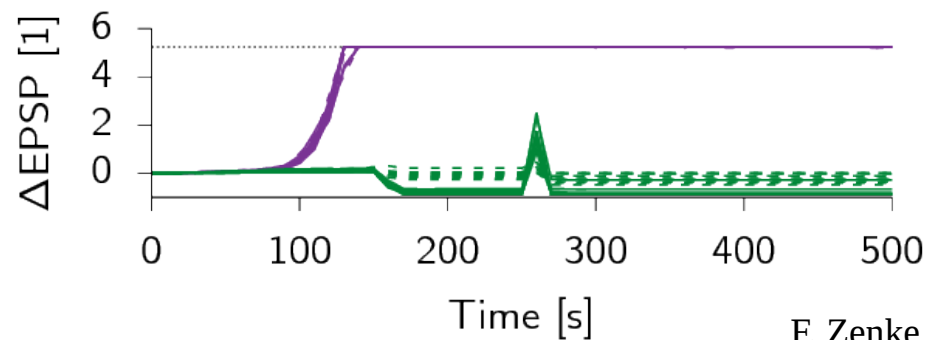
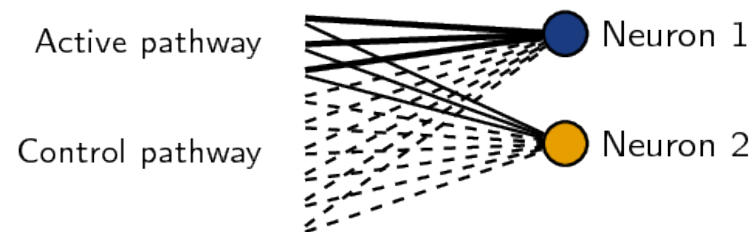
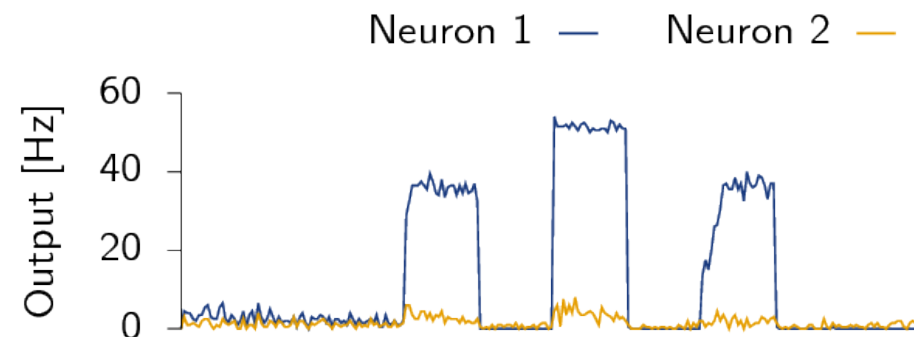
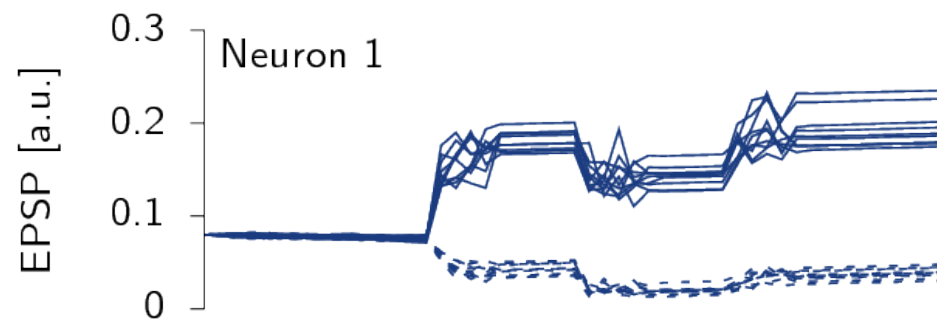
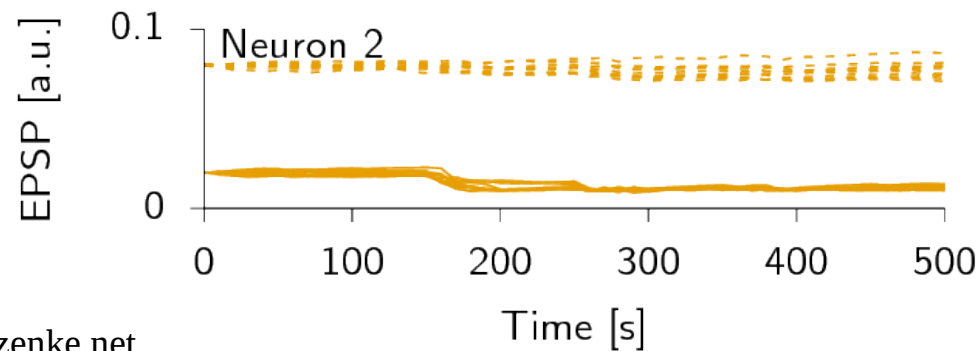


f

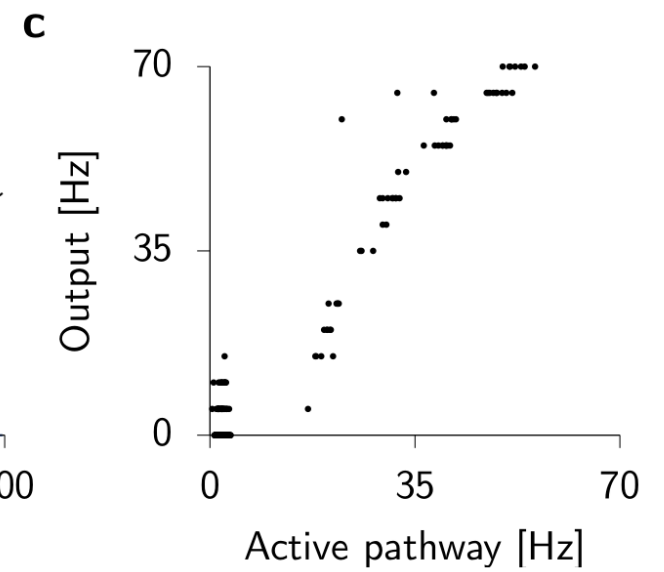
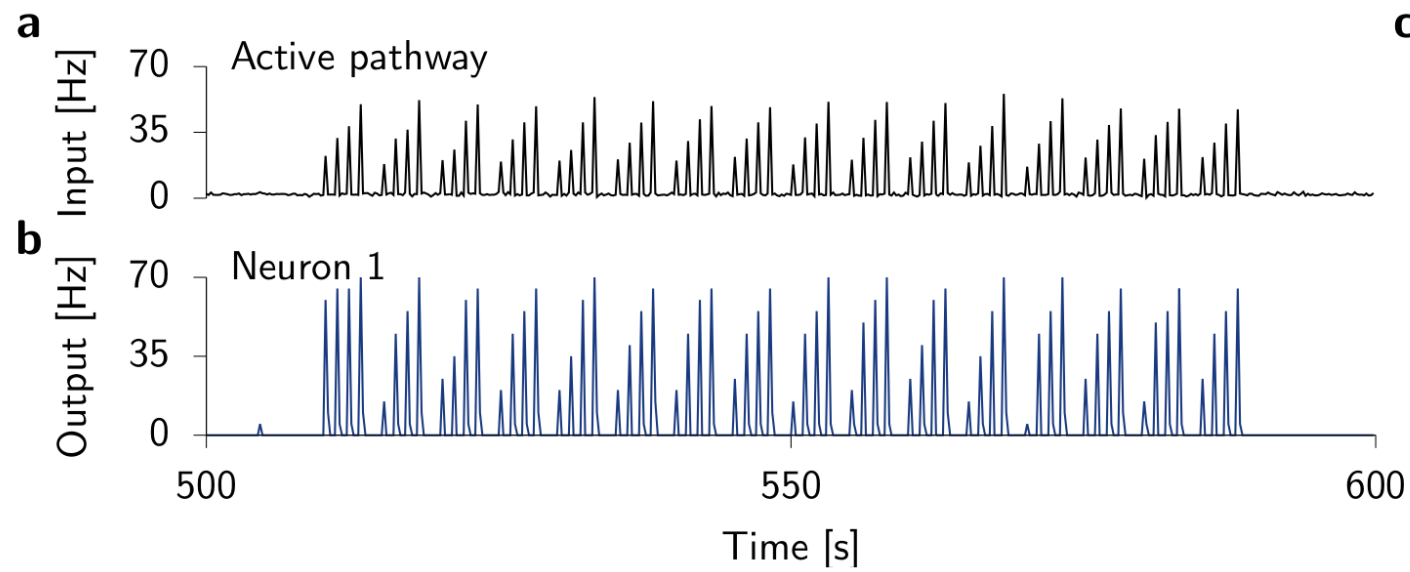


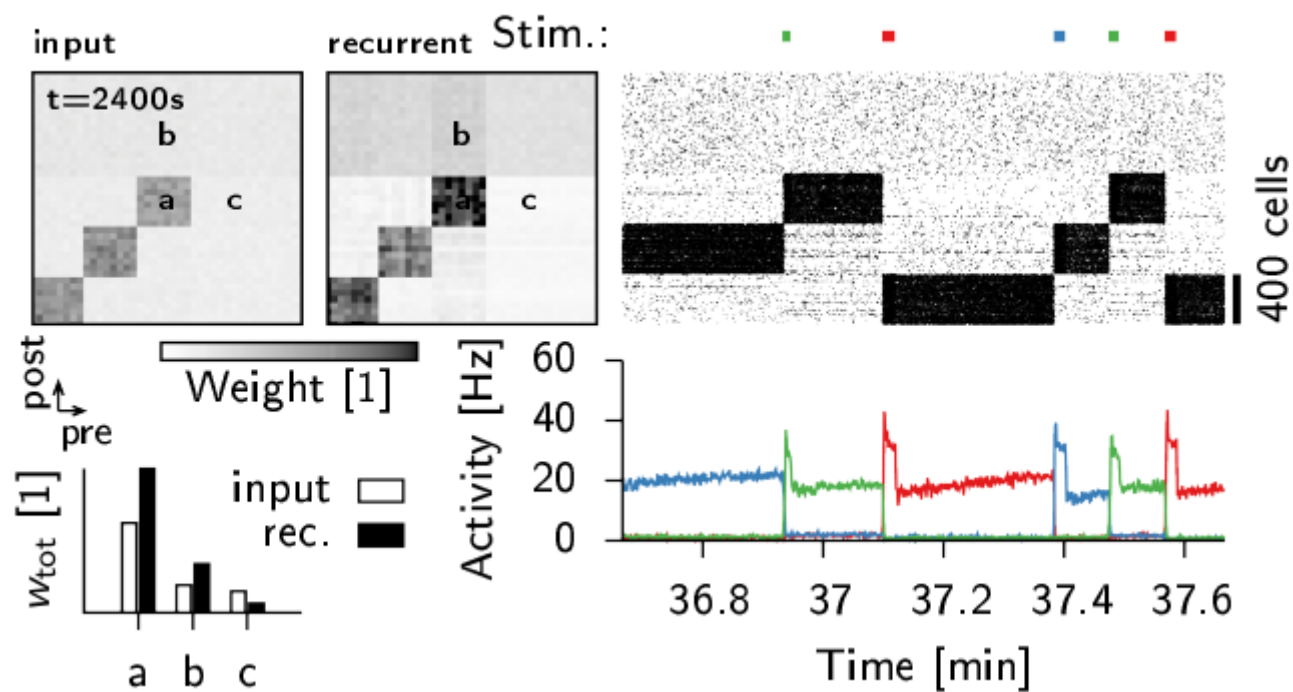
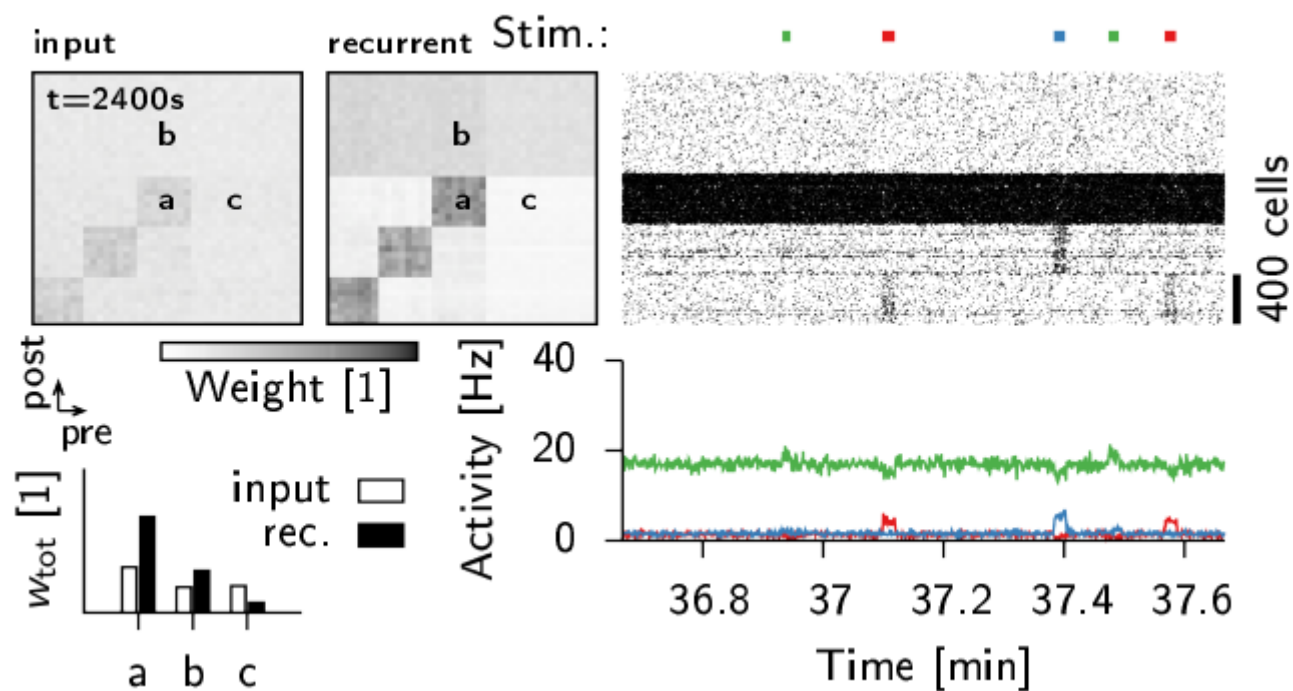
# Why Inhibitory Plasticity cannot solve all the problems

- LTP/LTD are not reversed by LTDi/LTPi  
→ edge effects
- ISP would have to be fast  
→ Experimental evidence suggests that ISP follows plasticity at excitatory synapses  
→ If it was fast: anti memories
- Presumably important for de-correlation (and balance)

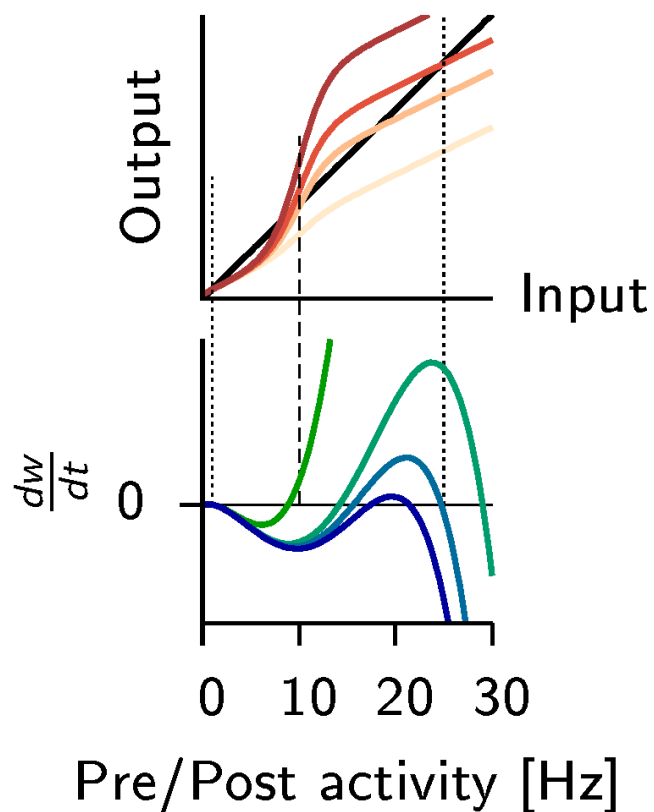
**a****b****c****d****e****f****g****h**





**a****b**

# How does this form of plasticity reveal itself in experiments?



- Visible in experiments in which (pre)=0
- Burst detector (only at high rates)
- Bi-directional

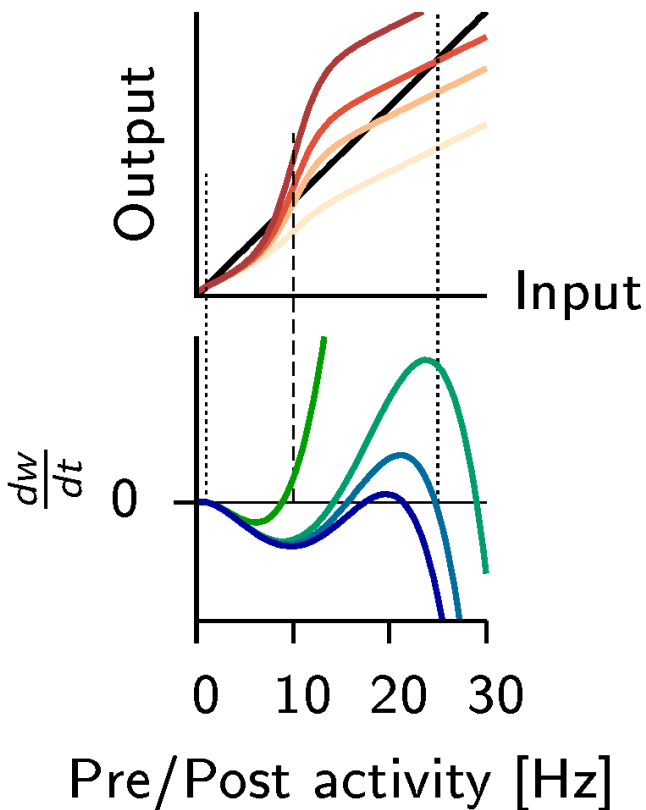
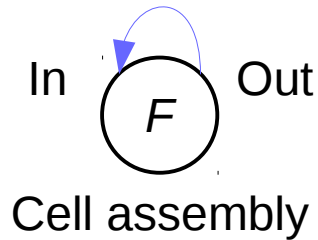
$$\frac{dw}{dt} \propto (\text{pre}) (\text{post}) ((\text{post}) - \kappa) \quad \text{Triplet}$$

$$- \beta (\text{post})^k (w - \tilde{w}) \quad \text{+Non-Hebbian}$$

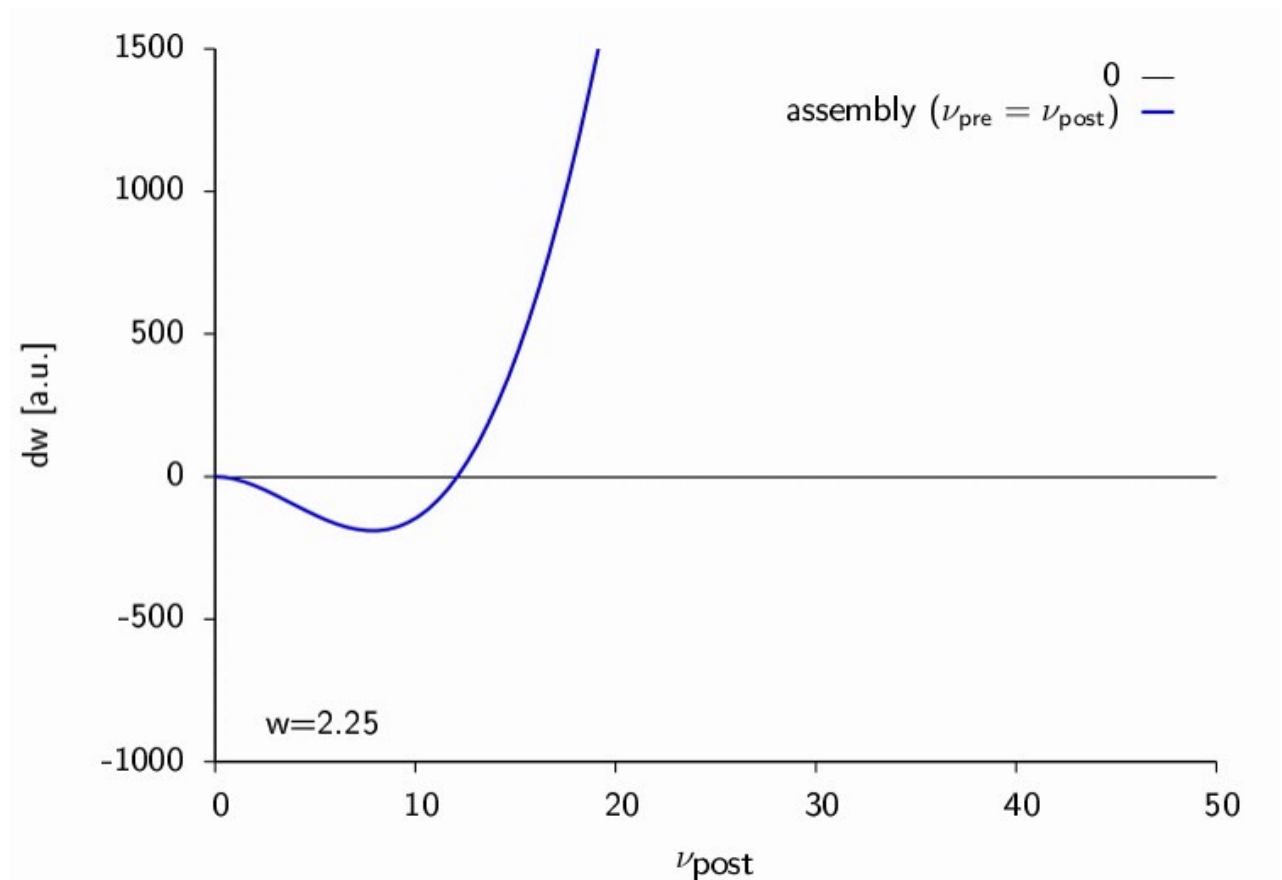
Heterosynaptic plasticity

$$k \geq 4$$

# Cell assembly fixed points self-tune to fixed points of network activity

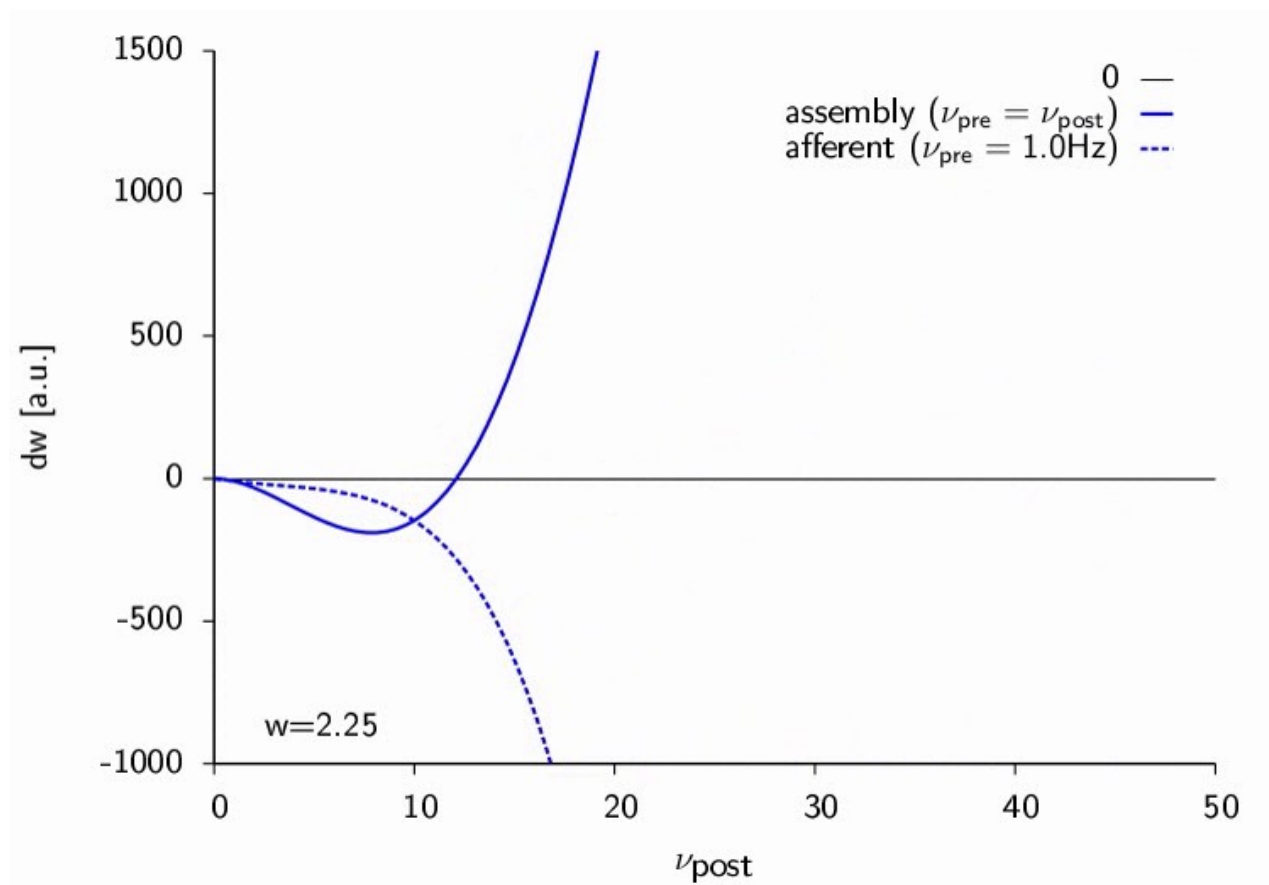
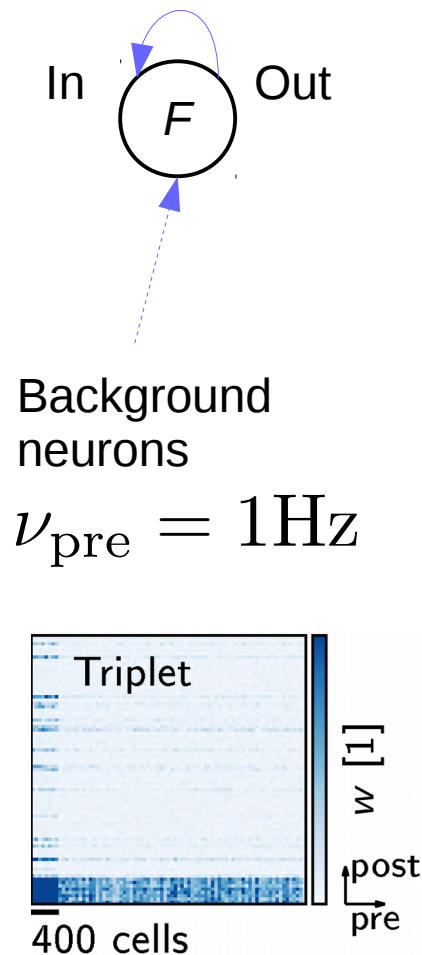


Zenke et al. (2015)



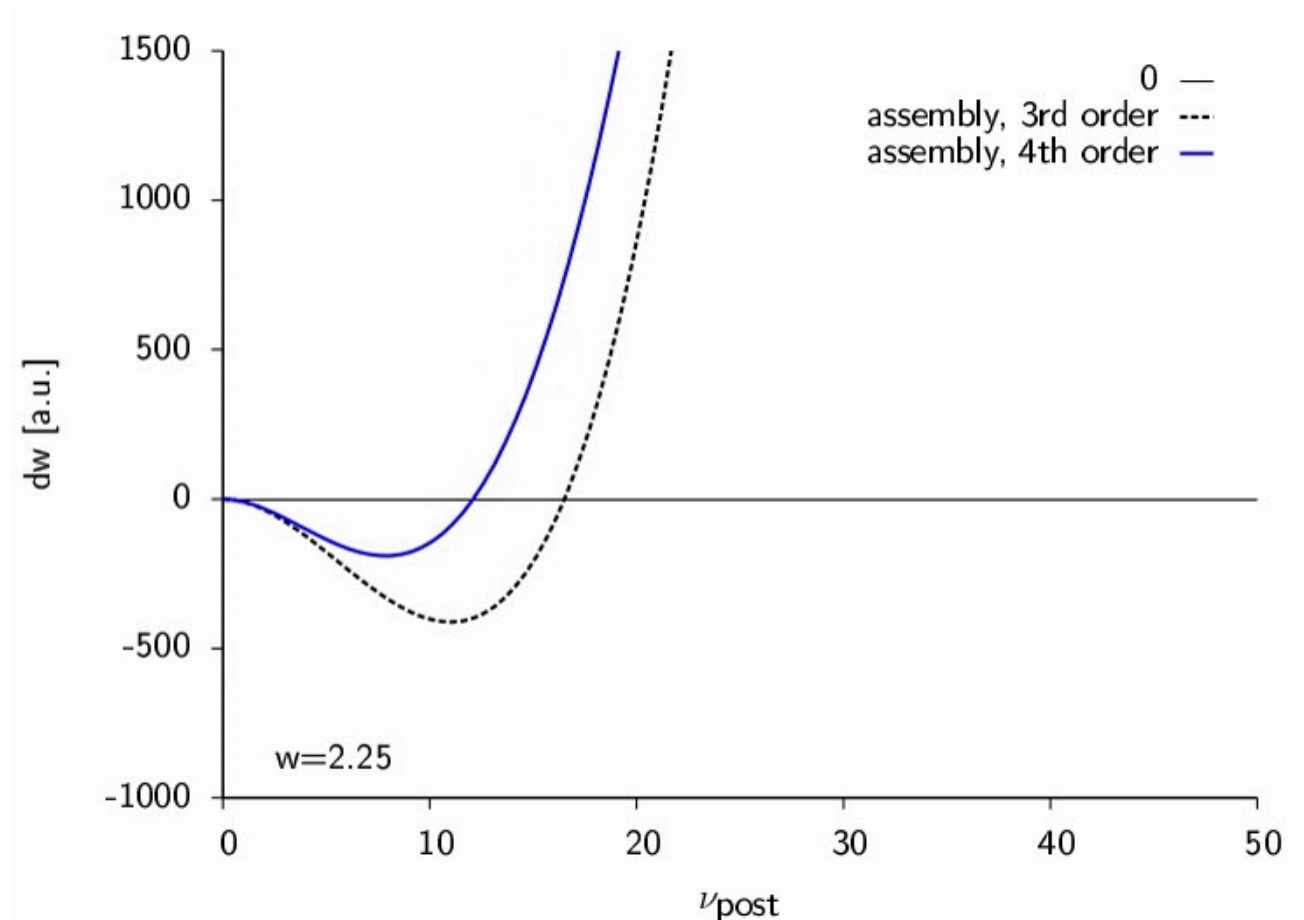
$$\frac{dw}{dt} \propto (\text{pre}) (\text{post}) ((\text{post}) - \kappa) - \beta (\text{post})^k (w - 1)$$

# Afferent connections increase little during assembly activation



$$\frac{dw}{dt} \propto (\text{pre}) (\text{post}) ((\text{post}) - \kappa) - \beta (\text{post})^k (w - 1)$$

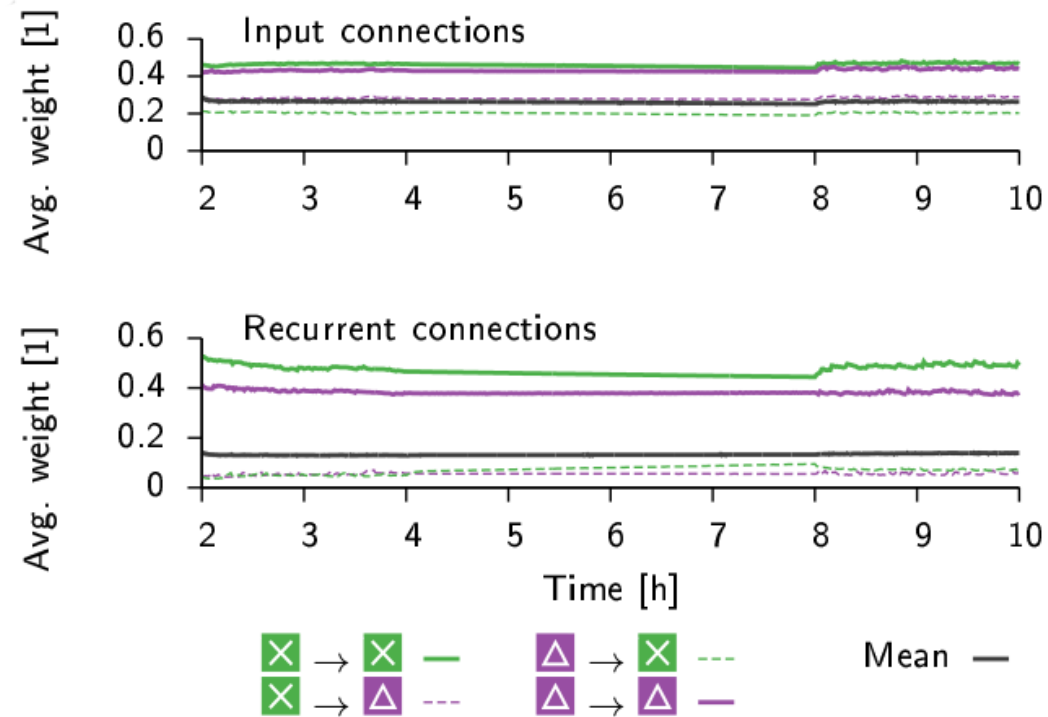
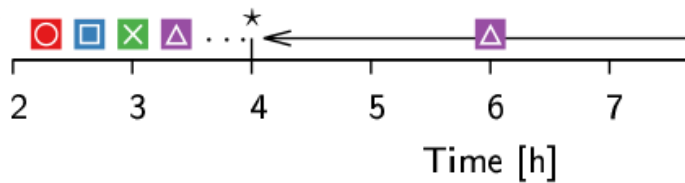
# Nonlinearity of heterosynaptic plasticity is crucial for bistability



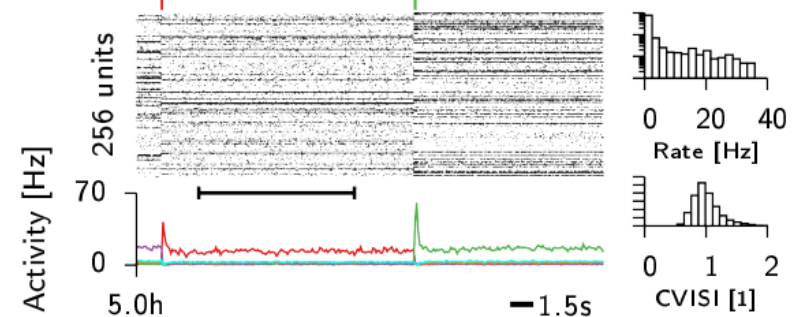
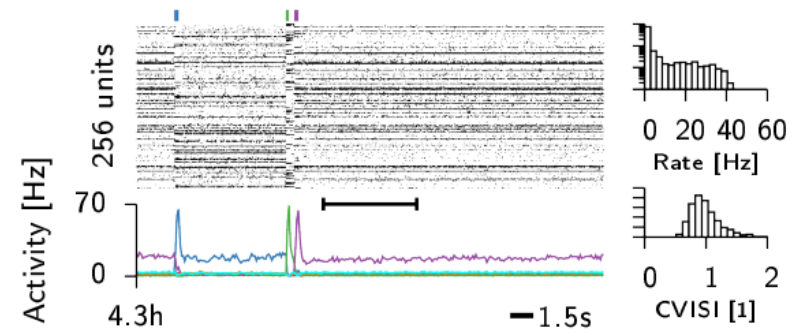
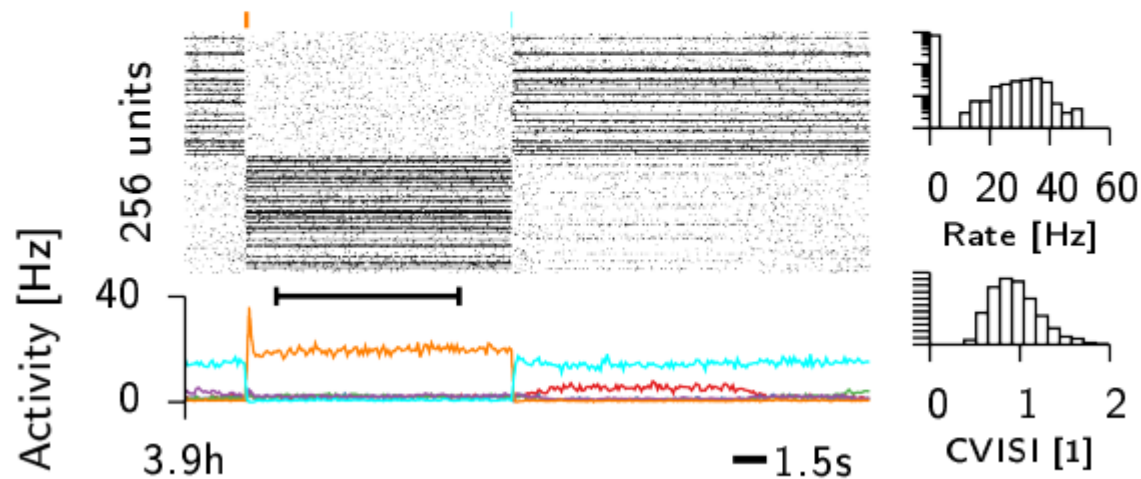
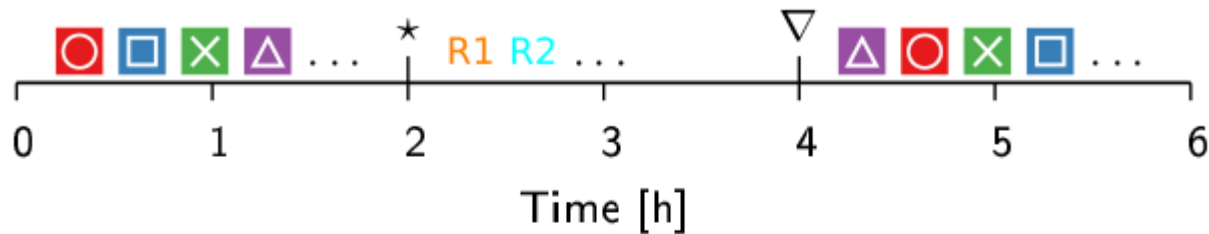
$$\frac{dw}{dt} \propto (\text{pre}) (\text{post}) ((\text{post}) - \kappa) - \beta (\text{post})^k (w - 1)$$

# Silent memories are stable

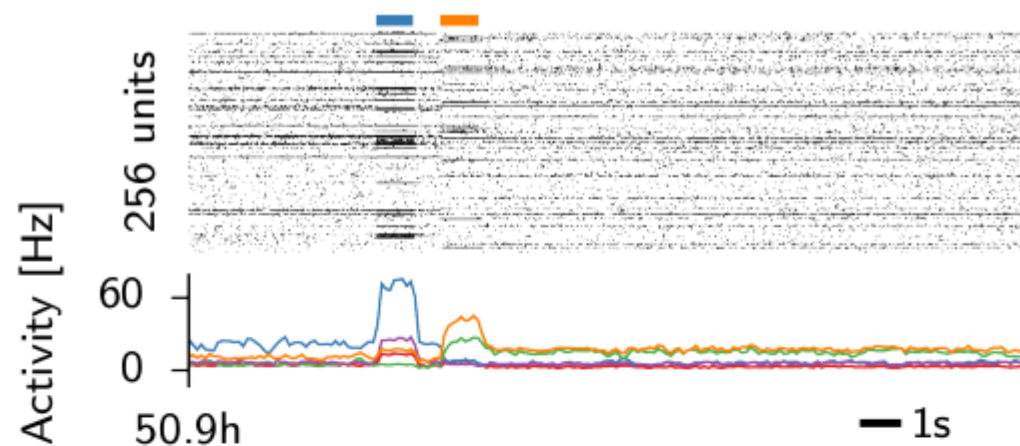
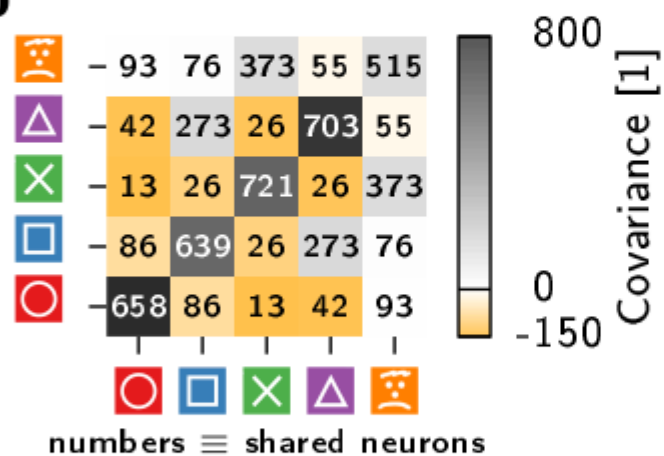
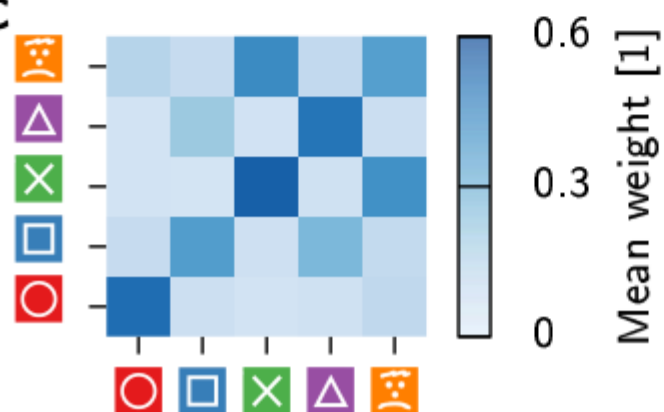
Learning



# Learning of new assemblies





**a****b****c****d**

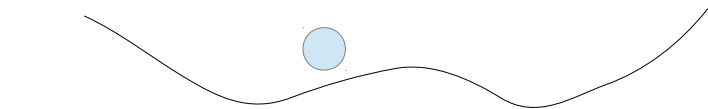
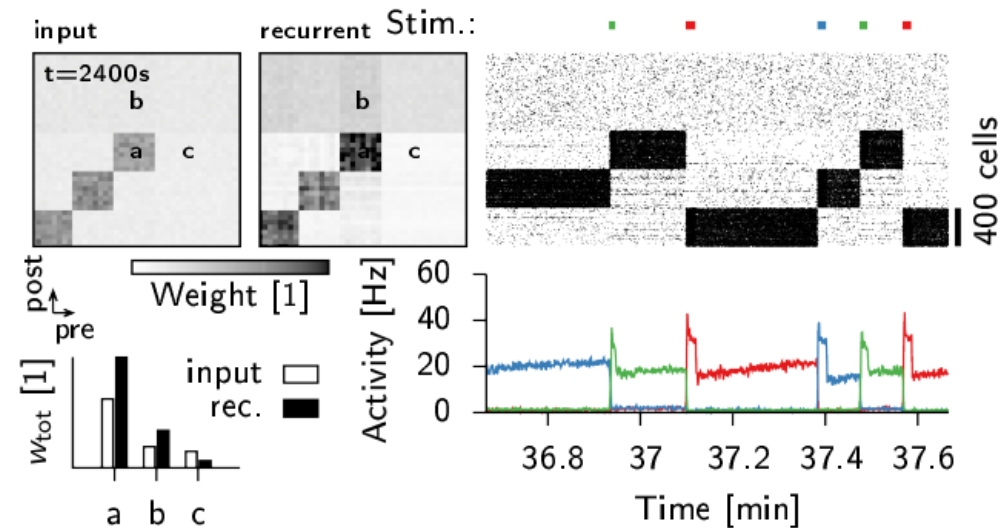
# Consolidation

$$\frac{dw}{dt} \propto (\text{pre}) (\text{post}) ((\text{post}) - \kappa) - \beta (\text{post})^k (w - \tilde{w})$$

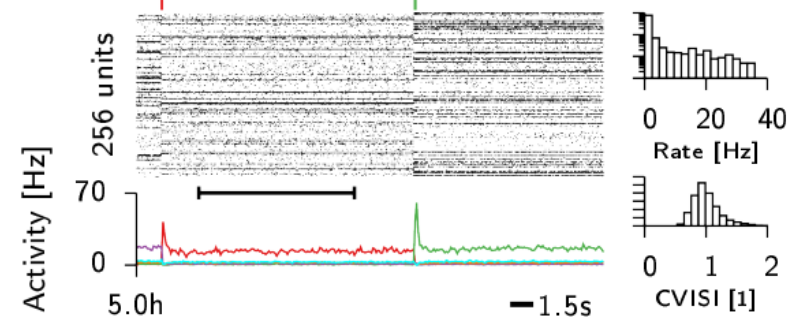
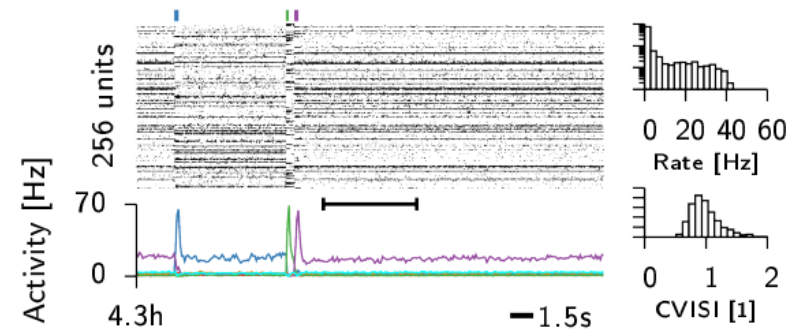
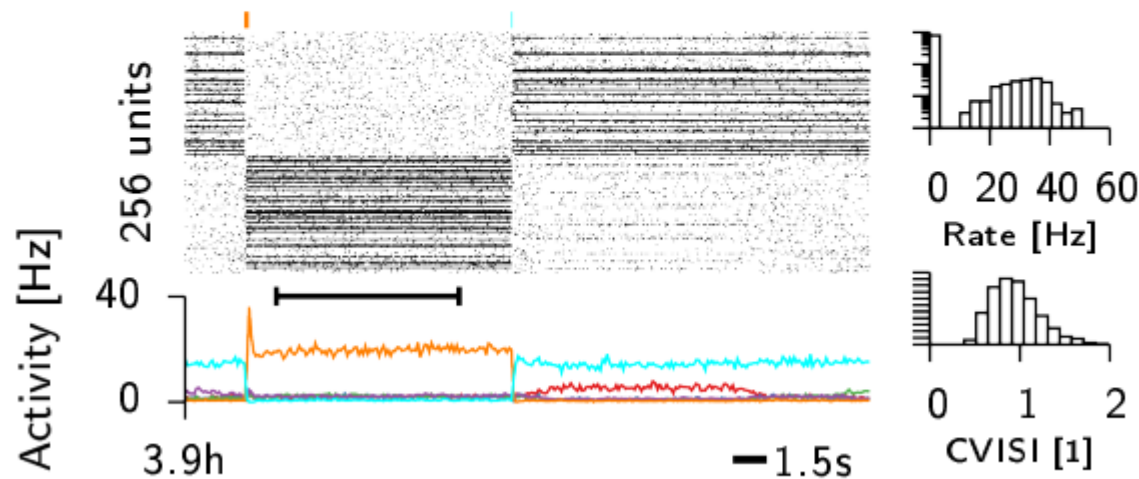
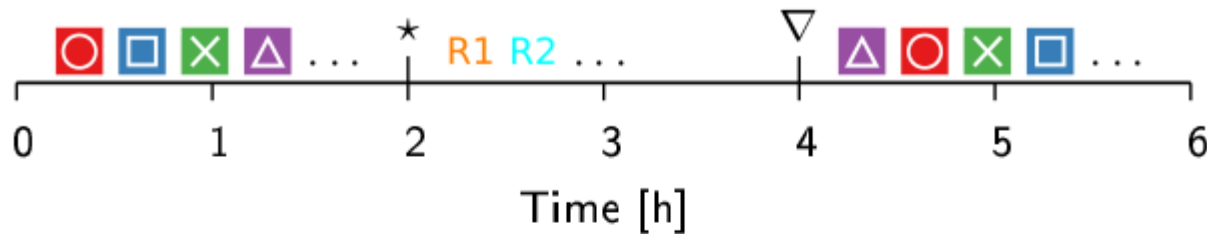
Slow consolidation dynamics

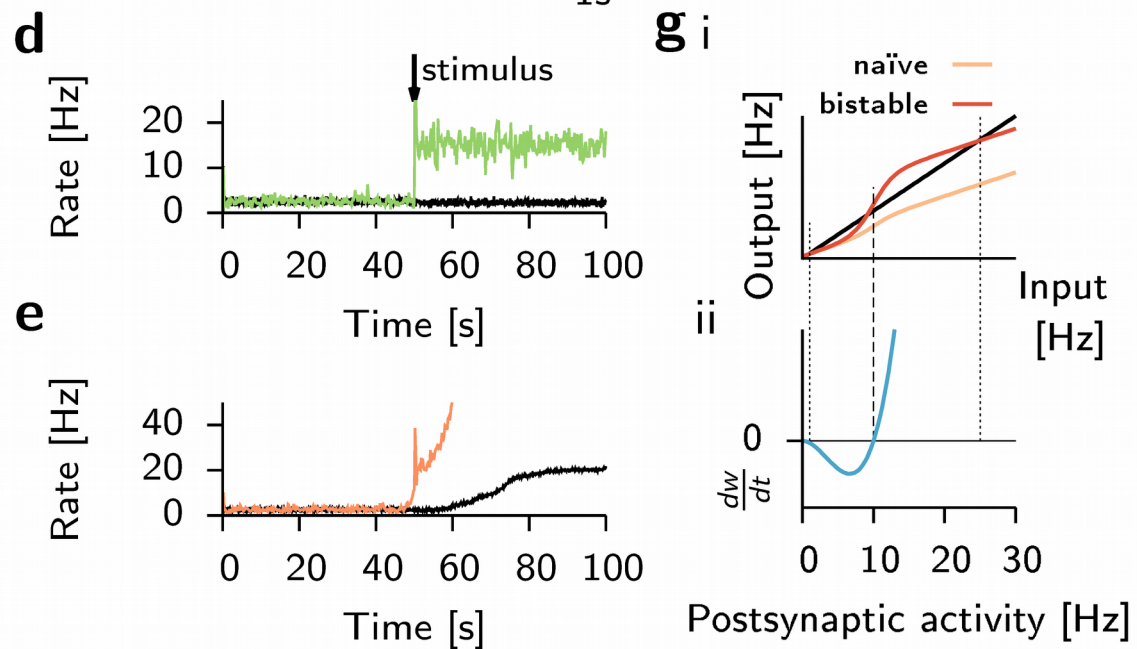
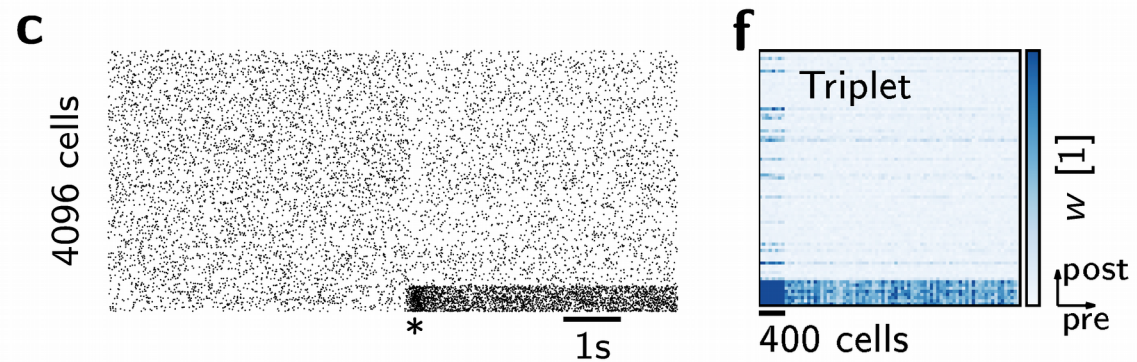
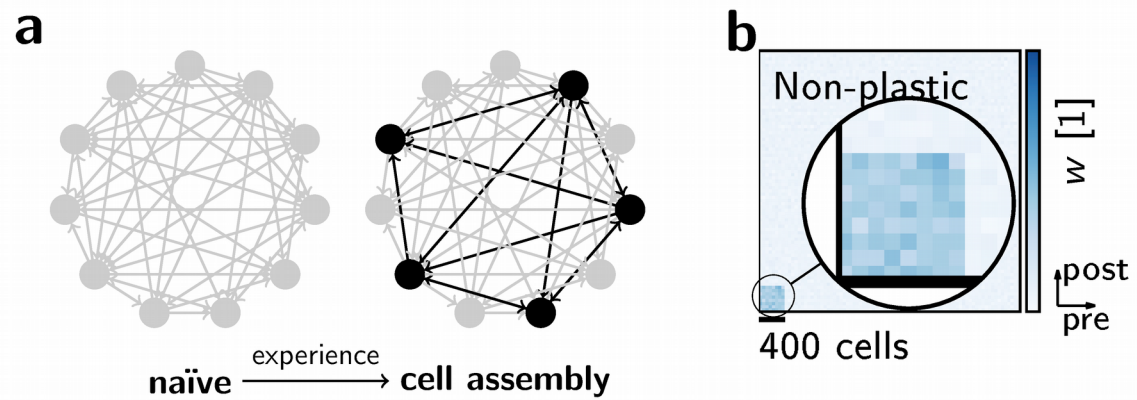
$$\tau_{\text{cons}} \frac{d\tilde{w}}{dt} = -\tilde{w} + w + f(\tilde{w})$$

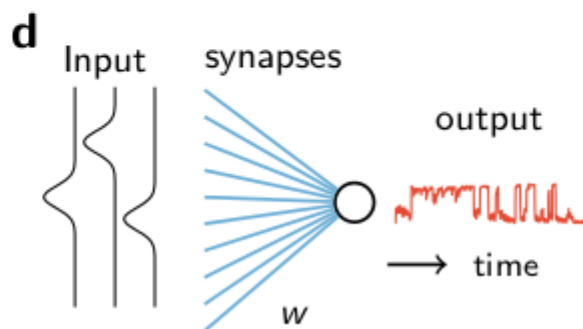
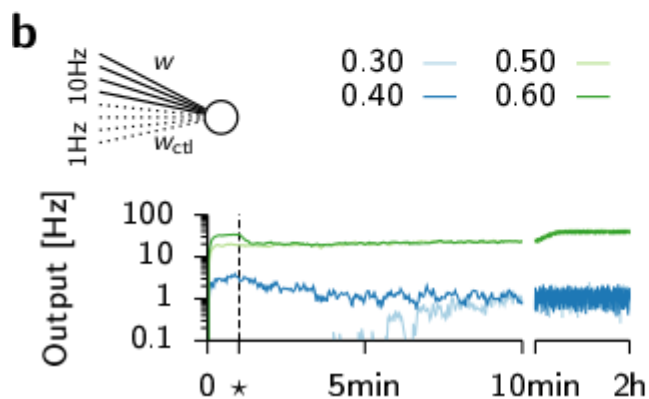
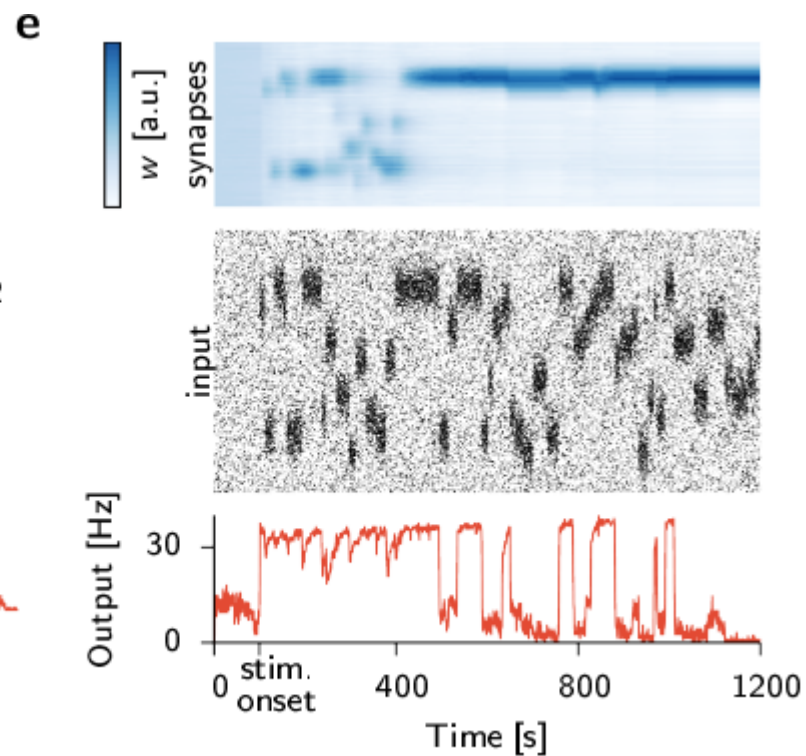
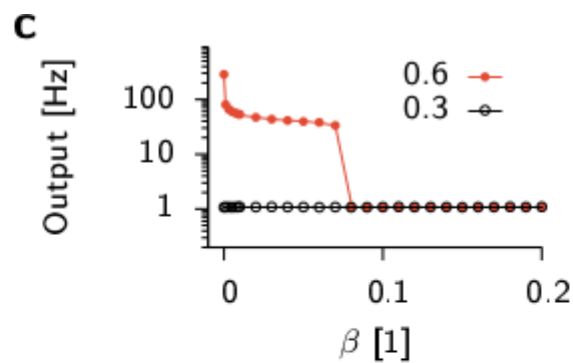
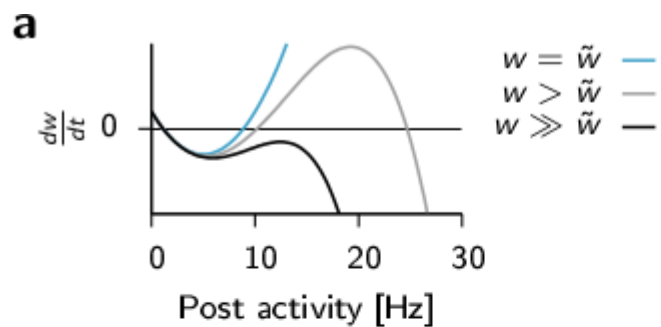
**a** With consolidation



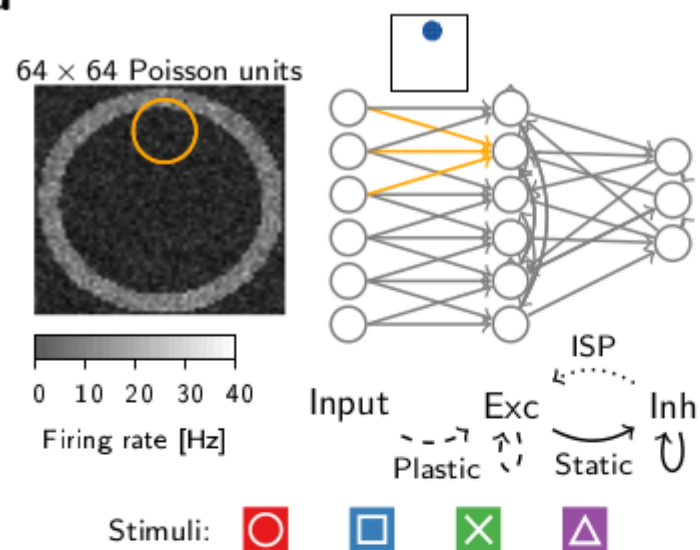
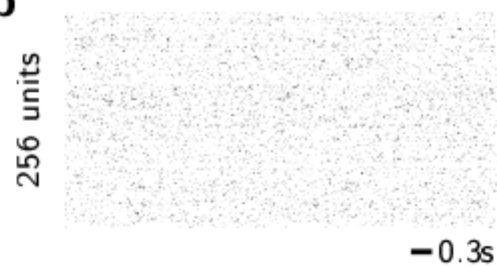
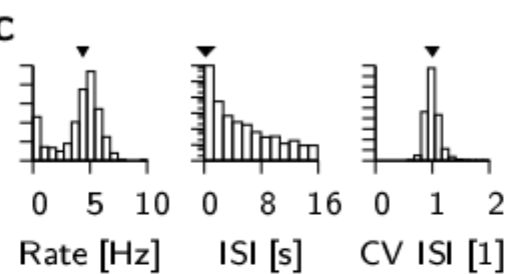
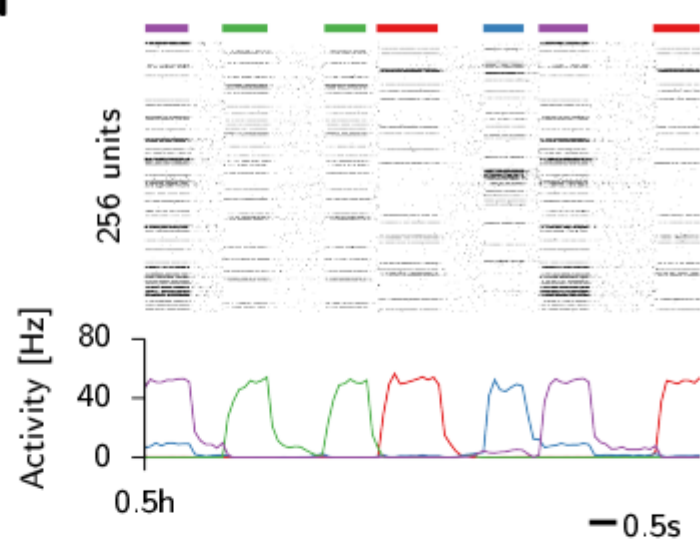
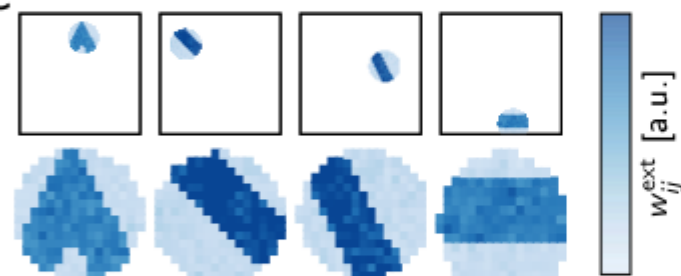
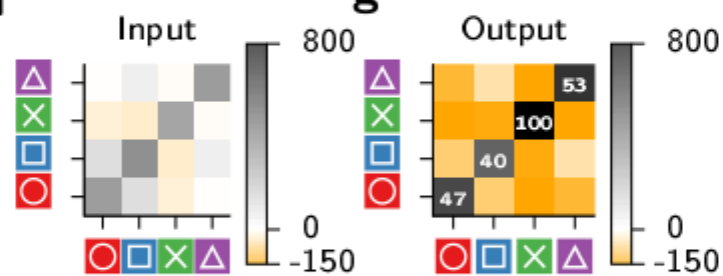
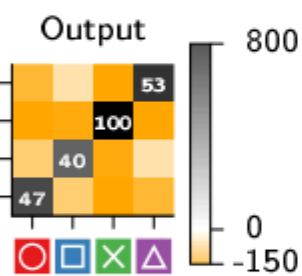
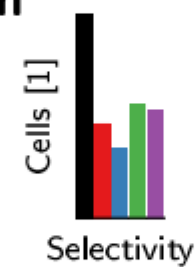
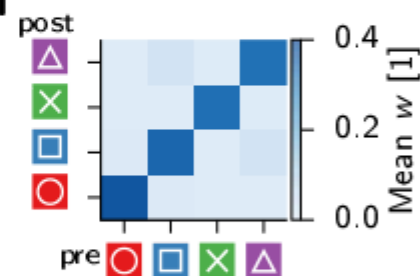
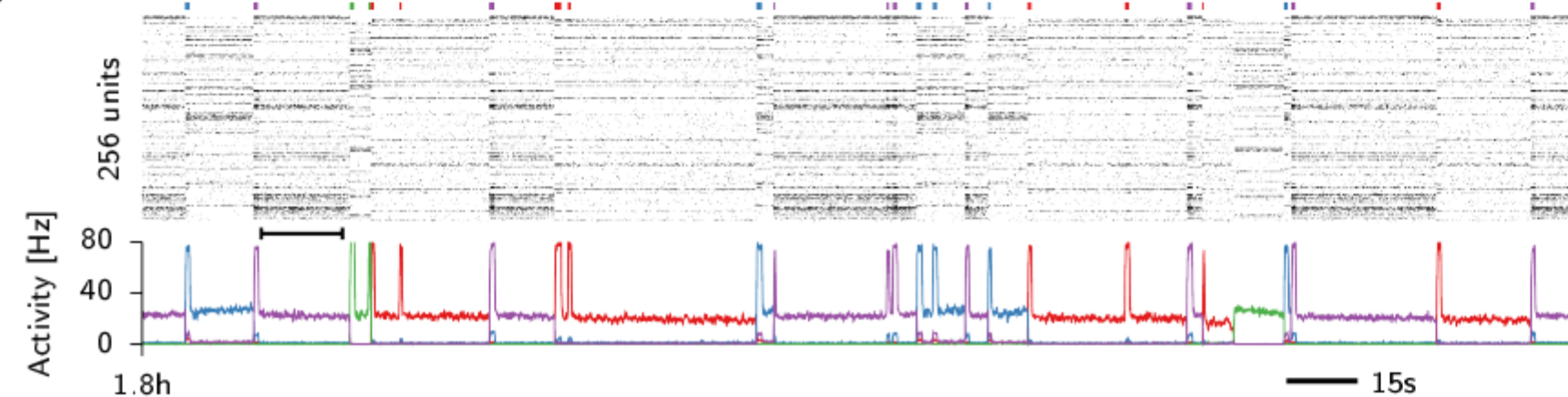
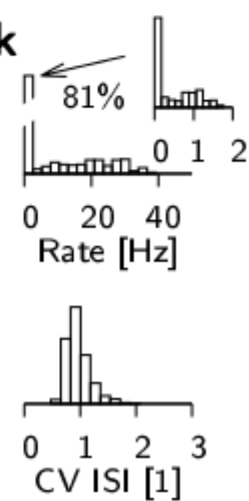
# Network stays plastic: Learning of new assemblies

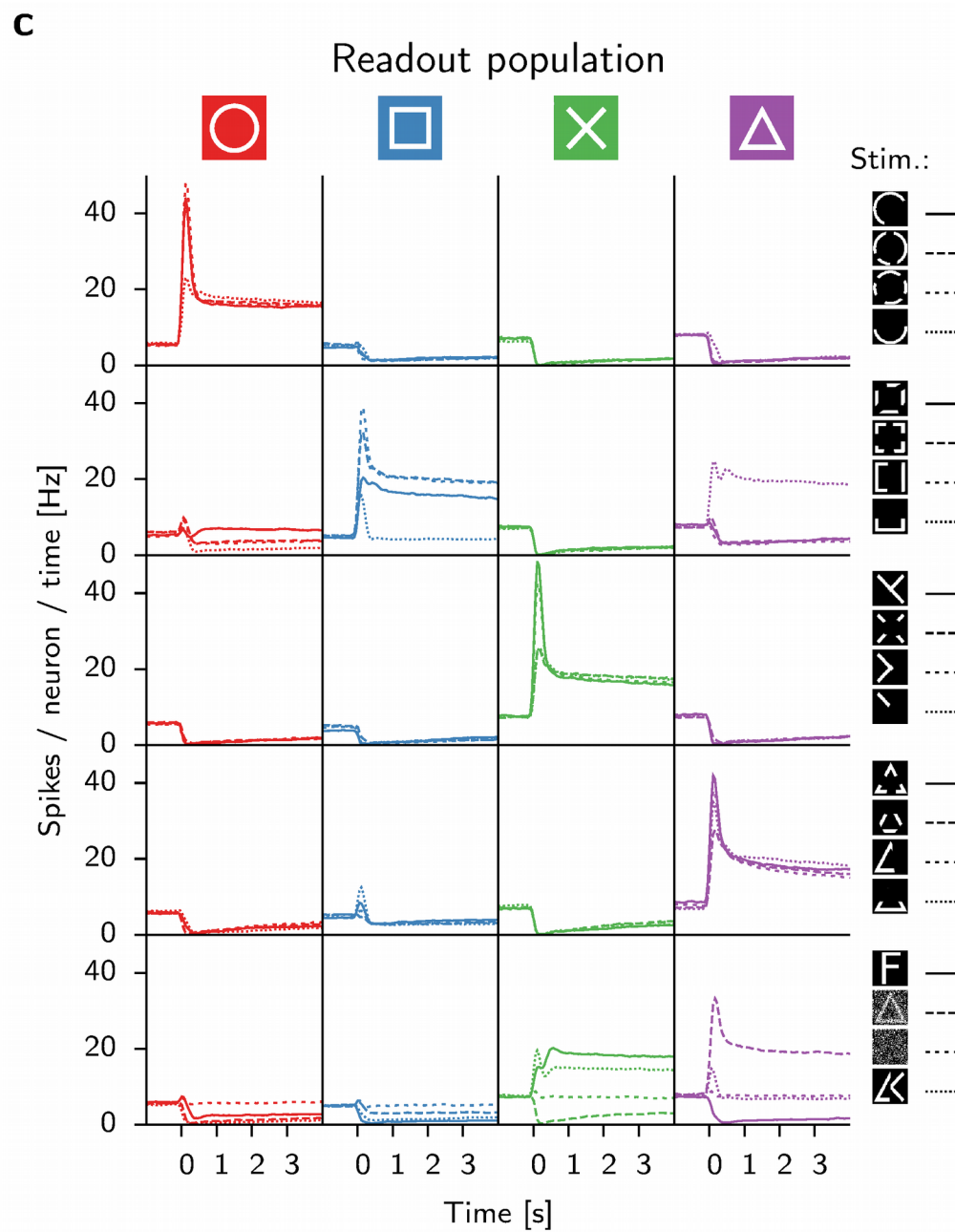
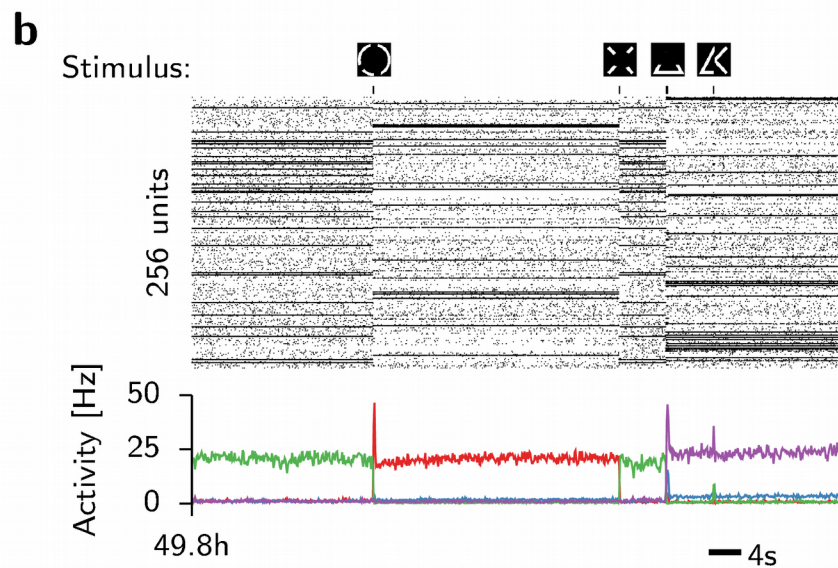
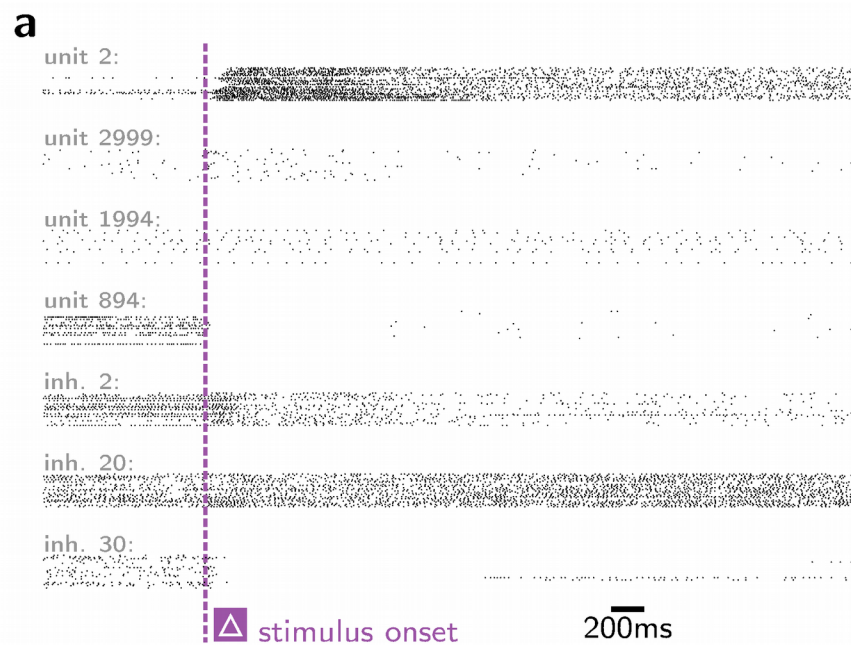


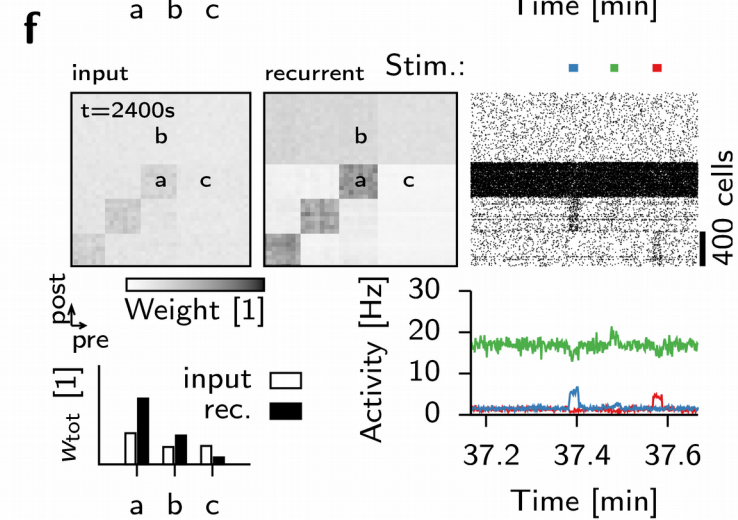
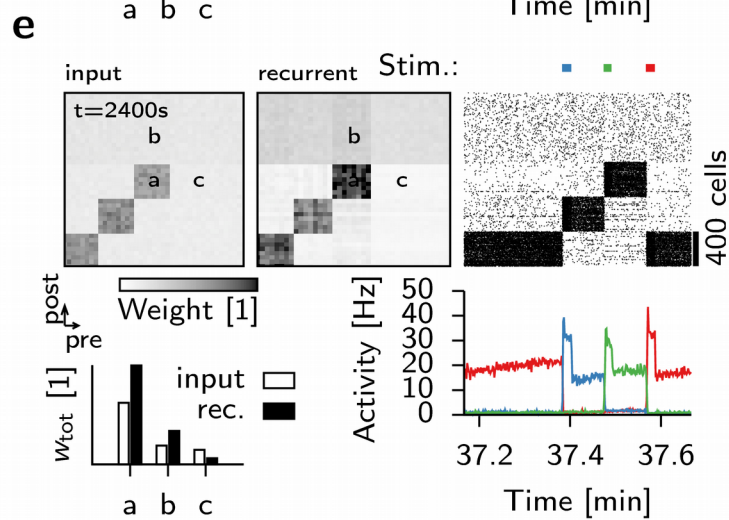
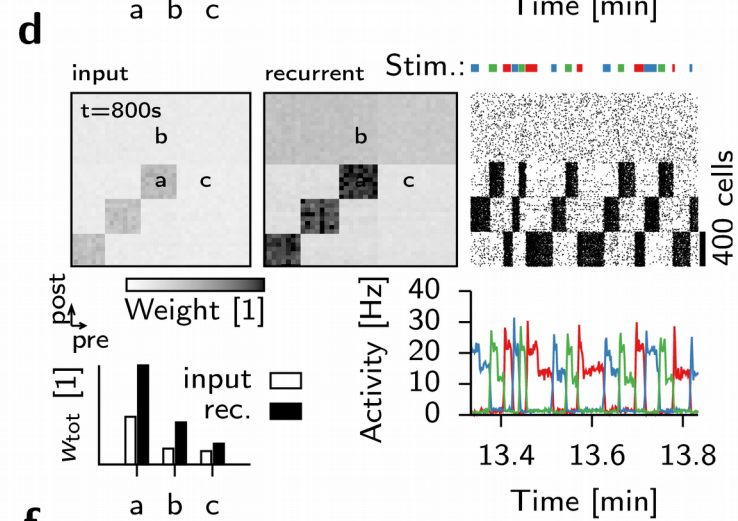
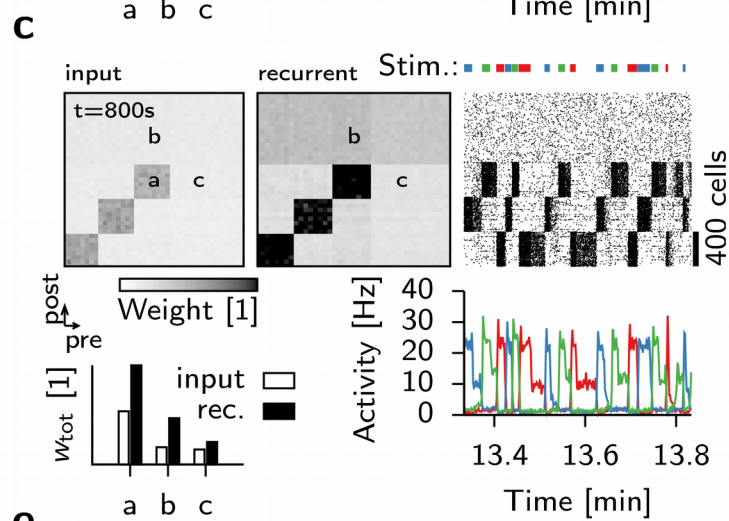
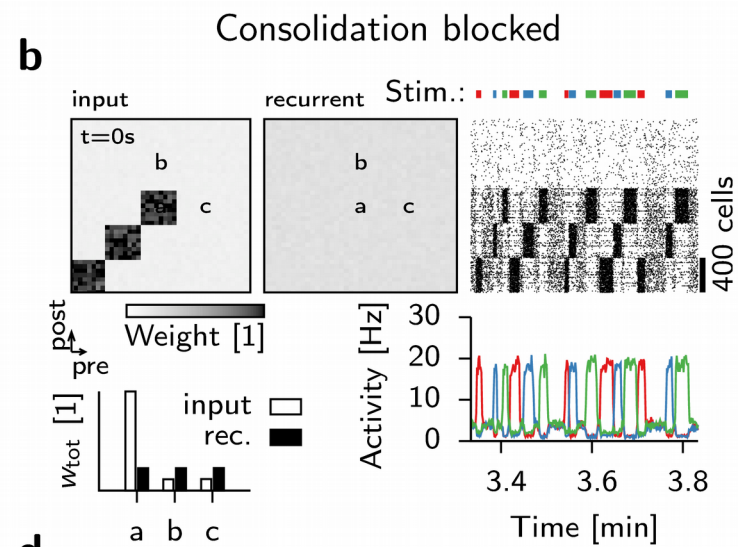
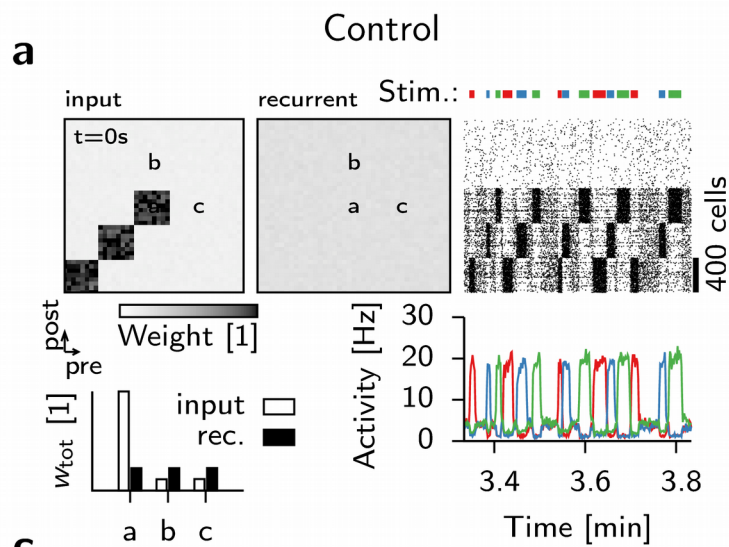






**a****b****c****d****e****f****g****h****i****j****k**







Weak initial state

No transmitter triggered

No heterosynaptic plasticity

No inhibitory plasticity

